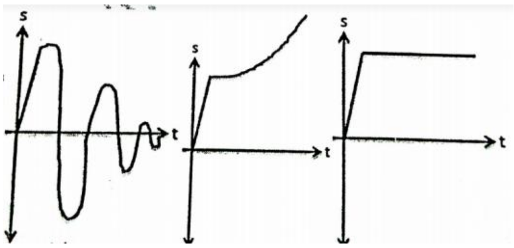
DYNAMIC STABILITY

The concept of dynamic stability is concerned with what actually happens to the mechanical system as a result of its displacement from the equilibrium condition.

1. Dynamic stability is related to the moving bodies, if any oscillations are generated by the system due to a disturbance and the amplitude of oscillations decreases continuously then the body is called as dynamically stable and the motion is called as damped oscillation (Graph-1).



2. If this system is statically unstable then the effect of small disturbance is to create a divergence from the equilibrium condition and these types of system are called as dynamically unstable (Graph-2).
3. If the system is statically neutral no oscillation are created neither the divergence, this system are called as dynamically neutral (Graph-3).
4. If the system is statically stable, dynamically it may be stable or may not be stable.
5. Dynamically stable ensures that the system is statically also stable.

NOTE: Static stability is pre-requisite for dynamic stability.

The equation for dynamic stability for a given airplane is known as characteristic equation or stability equation. The solution of the characteristic equation yields the Eigen values,

If Eigen value is purely real

1. A purely real Eigen value is associated with a **NON-OSCILLATORY MODE**, having a damping rate equal to the negative of the Eigen value.
2. If a real Eigen value is zero, the solution corresponds to a **RIGID BODY MODE**, describing a simple rigid body displacement.
3. If a real Eigen value is negative, the solution corresponds to a **CONNVERGENT MODE**, describing an exponential return to equilibrium following any disturbance.
4. If a real Eigen value is positive, the solution corresponds to a **DIVERGENT MODE**, describing an exponential deviation from equilibrium following any disturbance.

If Eigen value is Complex

1. A complex pair of Eigen value is associated with a **OSCILLATORY MODE**, having a damping rate equal to the negative of the real part of the Eigen value and a damped natural frequency equal to the imaginary part of the Eigen value.
2. If a real part of the Eigen value is zero, the solution corresponds to an **UNDAMPED MODE**, describing sinusoidal motion having constant amplitude.

<u>LONGITUDINAL DYNAMIC STABILITY</u>

Types of modes in longitudinal motion

THE SHORT PERIOD MODE

The Eigen value having the magnitude or largest imaginary part magnitude will always correspond to the short-period mode.

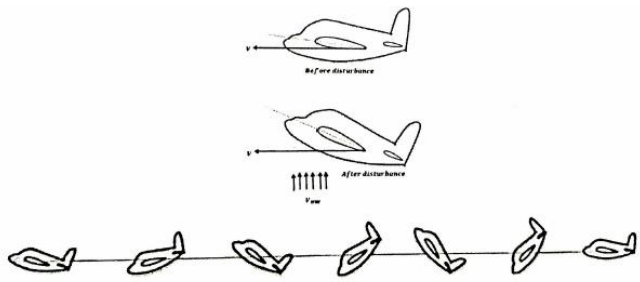
THE LONG PERIOD MODE (PHUGOID MODE)

The second largest Eigen value pair or smallest imaginary part magnitude will always correspond to the long period mode (Phugoid mode).

THE RIGID BODY MODE

These trivial rigid body modes simply states that the steady level flight can be maintained at altitudes.

A. *<u>DISTURBANCE IN INCIDENCE WITH NO CHANGE IN FORWARD SPEED – (SHORT PERIOD OSCILLATIONS)</u>*



Case – I: If the airplane is statically unstable:

If the airplane is statically unstable, i.e. $\frac{\mathrm{dCmcg}}{\mathrm{dC_L}} > 0$, then the effect of a vertical gust is to create a positive pitching moment resulting in overturning of an airplane and thereby increasing the angle of attack further, which will further increase the moment and the airplane may go out of control.

Case – II: If the airplane is statically neutral:

If the airplane is statically neutral, i.e. $\frac{\mathrm{dCmcg}}{\mathrm{dC_L}} = 0$, then neither restoring nor overturning moment is created and thus the airplane doesn't have any tendency to go back to the equilibrium position or to deviate further away from the equilibrium position but remains in the new displaced position, which represents dynamically neutral case.

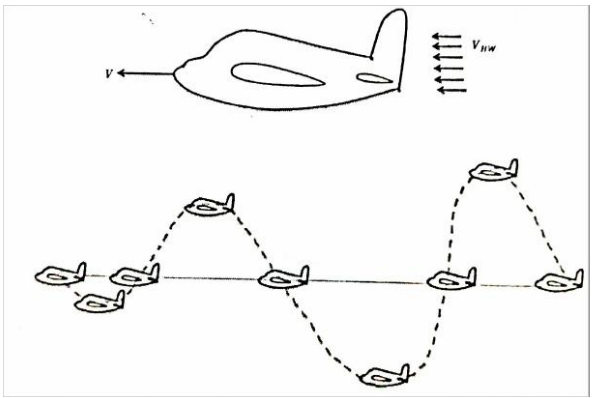
Case – III: If the airplane is statically stable:

If the airplane is statically stable, i.e. $\frac{\mathrm{dCmcg}}{\mathrm{dC_L}} < 0$, then it will try to regain its original equilibrium condition but in doing so, it may lead to the pitching oscillations about the equilibrium condition, these oscillations are called as short period oscillations.

NOTE:

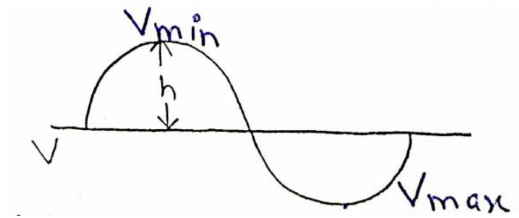
1. Generally in this mode, the oscillations will be damped within short period and therefore it has very high frequency.
2. Due to shortage of time, the pilot may not get sufficient time for correction and thud strong stabilizers should be used on the airplane.

B. *DISTURBANCE IN FORWARD SPEED WITH NO CHANGE IN INCIDENCE- (LONG PERIOD OSCILLATIONS/PHUGOID)*



1. If the airplane flying in steady state and level flight is subjected to a horizontal gust, then there will be a change in forward speed without change in incidence.
2. This change in forward speed will result change in aerodynamic forces such as lift and drag.
3. Consider a wind from tail side (disturbance) due to which the velocity of an airplane increases, therefore there is an increase in lift and drag and the airplane starts gaining height due to increment in lift.
4. During this, the kinetic energy is converted into potential energy and when it reaches the maximum altitude, it starts losing the height by exchanging its potential energy for kinetic energy.
5. This cycle may be repeated for number of times and this type of oscillations about the equilibrium condition, where, the airplane gains and loses the altitude with the loss and gain of velocity respectively is called as Phugoid motion.
6. This type of oscillations will be damped out after a long time.

DERIVATION FOR PHUGOID:



Let the mean velocity of an airplane be 'V' and the change in velocity due to horizontal gust be 'v', let 'h' be the change in height, therefore,

Change in potential energy, $\Delta PE = mgh$

Change in kinetic energy,

$$\Delta KE = \frac{1}{2}m(V+v)^2 - \frac{1}{2}m(V)^2$$

$$\Delta KE = \frac{1}{2}m(V^2+2Vv+v^2) - \frac{1}{2}mV^2$$

$$\Delta KE = \frac{1}{2}m(2Vv+v^2)$$

$\Delta KE = mVv$ (Neglecting v^2 being small compared to other terms)

But the total energy should be constant,

$$\therefore \Delta KE = \Delta PE$$

$$\mathrm{mvV} = \mathrm{mgh}$$

$$\mathbf{vV = gh}$$

The second order differential equation for PHUGOID motion is

$$\frac{d^2h}{dt^2} + 2\,\frac{g^2}{V^2}h = 0$$

$$\omega_n = \sqrt{2}\,\frac{g}{V}\ \mathrm{rad}/\mathrm{sec}$$

$$f_n = \frac{g}{\sqrt{2}\pi V}$$

$$T = \sqrt{2}\,\pi\,\frac{g}{V}$$

ROUTH'S DISCRIMINANT

The equation for longitudinal dynamic stability for a given airplane is known as characteristic equation or stability equation, given as,

$A\lambda^4 + B\lambda^3 + C\lambda^2 + D\lambda + E = 0$

Where, A, B, C, D, E are constants

To check if it is a longitudinal dynamic stability equation, following points should be noted-

1. All the coefficients should be positive
2. B and C should be greater than D and E.
3. A should be almost or exactly equal to one.

e.g. $\lambda^4 + 5.05\lambda^3 + 13.2\lambda^2 + 0.67\lambda + 0.59 = 0$

Routh's Criteria:

The Routh's constant 'R' is given as,

$$R = BCD - D^2A - B^2E$$

CASE- I: R>0

- i. It shows dynamically stable condition.
- ii. Roots of the characteristic equations will be complex pairs with negative real parts.
  e.g. (-P1±iQ1) and (-P2±iQ2)
- iii. All the modes are periodic and damped.
- **iv. It generally has two modes, i.e. Phugoid and Short period oscillations.**

CASE- II: R = 0

It represents dynamically neutral condition

CASE- III: R < 0

- i. It shows dynamically unstable condition; in this case one of the roots having complex pair with positive real part represents un-damped oscillations.
  e.g. (+P1±iQ1)
- ii. The other pair represents a Phugoid or short period oscillations.

CASE- IV: E = 0

- i. One of the values of λ is zero, which shows one mode is dynamically unstable.
- ii. If other roots are negative, then it shows pure convergence.
- iii. If other roots are positive, then it shows pure divergence.

CASE-V: If any coefficient in the characteristic equation has a negative sign, one of the modes will give pure convergence or dynamically unstable condition.

TIME PERIOD (T)

$$T = \frac{2\pi\hat{t}}{|q|}$$

Where, $\hat{t} = \frac{m}{\rho V S}$, i .e. airplane density factor

M= mass of airplane

ρ = free sream density

V = free stream velocity

S = wing area

q = imaginary part of root

p = real part of root

TIME TO HALF ($T_{1/2}$):

$$T_{1/2} = \frac{-0.693\hat{t}}{p}$$

NUMBER OF CYCLES TO HALF ($N_{1/2}$):

$$\boldsymbol{N_{1/2} = f_n \times T_{1/2}}$$

Where, $f_n = \frac{\omega_n}{2\pi}$

PROBLEM: The quartic of longitudinal dynamic of an airplane is given as

$(\lambda^2 + 5.0159\lambda + 12.984)(\lambda^2 + 0.03406\lambda + 0.0454) = 0$

Calculate the time period, time to half, number of cycles to half, identify the modes and compute the damping factor and natural frequency of the modes.

SOLUTION:

Considering,

$$(\lambda^2 + 5.0159\lambda + 12.984) = 0$$

$$\lambda_{1,2} = -2.508 \pm i2.587$$

And $(\lambda^2 + 0.03406\lambda + 0.0454) = 0$

$$\lambda_{1,2} = -0.017 \pm i0.2125$$

Both the roots are complex with negative real part (representing damped oscillatory motion)

$\lambda_{1,2} = -2.508 \pm i2.587$ With higher magnitude represents short period oscillations (Higher value of damping).

Comparing with $-\xi\omega_n \pm i\sqrt{1-\xi^2}\omega_n$

We get, $\xi = 0.6961$ *and* $\omega_n = 3.6029\frac{rad}{s}$

$\lambda_{1,2} = -0.017 \pm i0.2125$ With lower magnitude represents Phugoid (Lower value of damping).

Comparing with $-\xi\omega_n \pm i\sqrt{1-\xi^2}\omega_n$

We get, $\xi = 0.0798$ *and* $\omega_n = 0.213\frac{rad}{s}$

TIME PERIOD (T):

$$\text{T(short period)} = T = \frac{2\pi\hat{t}}{2.587} = 2.4287\hat{t}$$

$$\text{T(Phugoid)} = T = \frac{2\pi\hat{t}}{0.2125} = 29.56\hat{t}$$

Hence, Phugoid motion will take more time then Short period oscillations to return to the equilibrium position.

Time to Half ($T_{1/2}$):

$T_{1/2}$(short period) = $\frac{-0.693\hat{t}}{2.508} = 0.2763\hat{t}$

$T_{1/2}$(Phugoid) = $\frac{-0.693\hat{t}}{0.017} = 40.76\hat{t}$

Number of cycles to half ($N_{1/2}$):

$N_{1/2}$(short period) = $0.1584\hat{t}$

$N_{1/2}$(Phugoid) = $1.3817\hat{t}$

TYPES OF MODES IN LATERAL MOTION

The Eigen values corresponding to the lateral motion of a typical airplane contains two distinct real roots, one complex pair and two identical zero roots.

ROLL MODE AND SPIRAL MODE

The purely real and distinct roots corresponds to two non oscillatory odes i.e. Roll mode and spiral mode. The larger Eigen value correspond to highly damped motion known as **ROLL MODE** while the smaller one corresponds to the lightly damped or divergent motion called the spiral mode.

DUTCH ROLL MODE

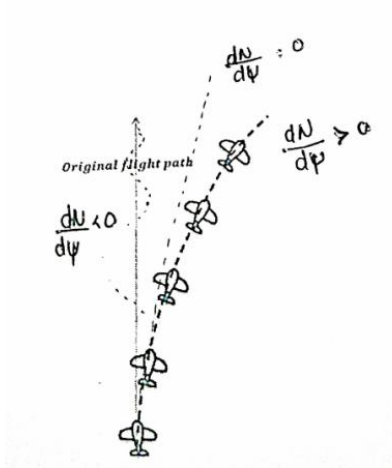
The complex pair describes the oscillatory motion called **DUTCH ROLL** $(-\xi\omega_n \pm i\sqrt{1-\xi^2}\omega_n)$

RIGID BODY MODE

The remaining two zero Eigen values corresponds to rigid body displacements.

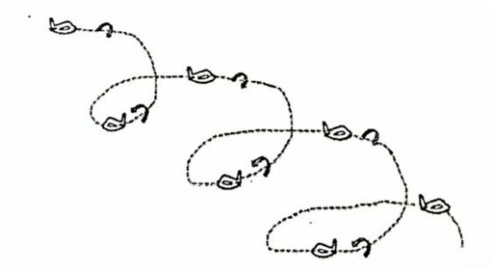
DYNAMIC EFFECTS

A. DIRECTIONAL DIVERGENCE:



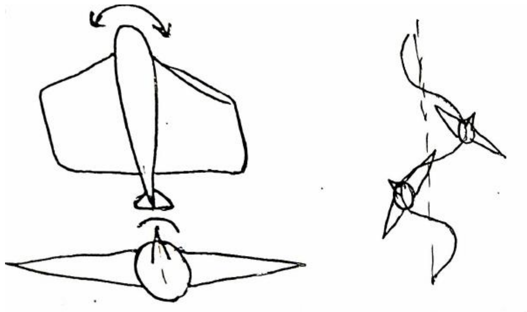
1. If an airplane is directionally unstable, i.e.$\frac{dN}{d\psi} > 0$), a positive change in angle of yaw creates a positive yawing moment.
2. If an airplane is flying in a steady state and level flight and experiences a small disturbance in yaw to the starboard side, the result will be a yawing moment to the starboard side as an airplane is directionally statically unstable.
3. This will cause the deviation to increase and in the resultant yawed attitude, an airplane will keep moving more and more away from the equilibrium condition (original flight path), and this type of motion is called as directional divergence.

B. SPIRAL DIVERGENCE:



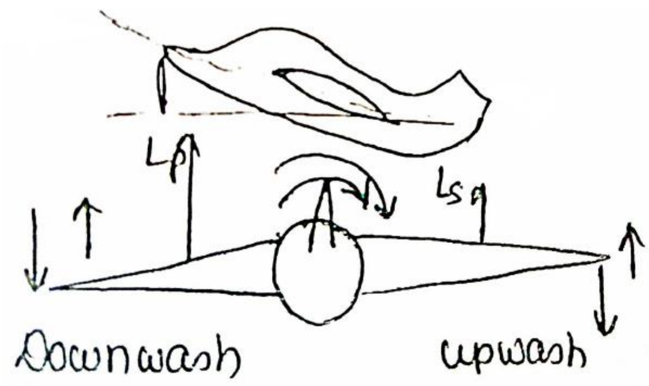
1. If an airplane is directionally as well as laterally statically unstable, i.e. ($\frac{dN}{d\psi} > 0$) as well as ($\frac{dL}{d\Phi} > 0$).
2. If an airplane yawed to the starboard side, a positive yaw creates positive yawing moment and the airplane starts deviating more and more away from an equilibrium path.
3. But the yawing doesn't start alone; yawing to the starboard also initiates rolling to the starboard due to cross coupling effect.
4. AS an airplane is laterally also unstable, positive roll creates positive rolling moment and therefore airplane keeps rolling to the starboard side.
5. As an airplane rolls, the lift vector is titled and LcosΦ is less than the weight of an airplane, due to which it starts losing an altitude.
6. In the resultant motion, an airplane will therefore enter a spiral dive which may get tighter and tighter in accordance with insufficient stability; this kind of motion is called as spiral divergence.

C. DUTCH ROLL:



1. If an airplane is directionally statically stable as well as laterally statically stable, i.e. ($\frac{dN}{d\psi} > 0$) as well as ($\frac{dL}{d\Phi} < 0$).
2. A disturbance in yaw will result in oscillations in yaw, but we know, yawing doesn't occur alone and rolling is also initiated due to cross coupling effect.
3. As an airplane is laterally also stable, it will result in combinations in roll.
4. Therefore, the resultant motion will be a combination of oscillations in yaw and oscillations in roll with gradual reduction in altitude and this type of motion is called as Dutch roll.

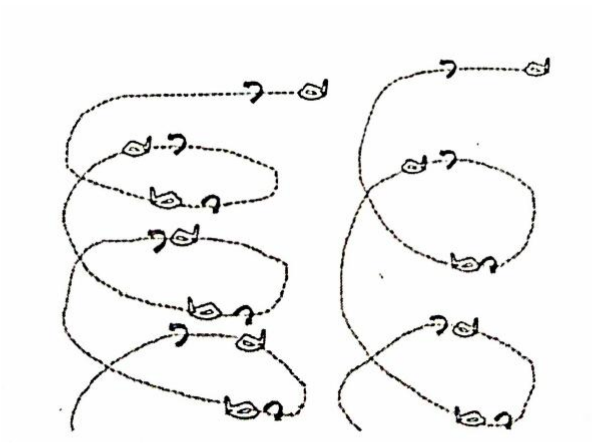
D. AUTOROTATION:



1. Consider an airplane flying very close to the stalling angle (α_{stall}), in this condition if an airplane is subjected to a roll to the starboard side, i.e. a initial roll.
2. The effective incidence of starboard wing increases due to up-wash and that of the port wing decreases due to down-wash.
3. This causes stalling of the star board wing resulting in loss of lift, on the other hand, lift of port wing is still high.
4. Due to this difference in lift of both the wings, an additional rolling moment will be created, which is in the same sense as an initial roll.

5. Thus an airplane will accelerate in roll and this phenomenon is known as AUTOROTATION.

E. SPIN:



1. In practical flight, an autorotation doesn't occur alone, it also initiates directional disturbance, and i.e. yaw due to cross coupling effect and the resultant motion is known as SPIN.
2. Let at high angle of attack, very close to the stalling angle, an airplane is rolled to the starboard side so that the starboard wing will stall.
3. The rolling moment also initiates yawing moment to the port side and thus there is an interaction between yawing due to roll and rolling due to yaw and a steady state condition is reached called as spin.
4. High incidence well beyond the stall results in flat spin in which rate of rotation is very high but descend is not very steep.
5. If spin occurs at relatively lower incidence, though an incidence must be high enough to cause one wing to stall, then the resultant spin is called as steep spin in which rate of rotation is not very high but the descend is very steep.

NOTE: Phugoid and short period oscillations are also two modes of Dynamic effects.

<u>AERO-ELASTIC EFFFECTS</u>

Introduction

1. An airplane structure is not perfectly rigid; an airframe is always subjected to the aerodynamic loads which tend to modify the shape of the structure.
2. The change in shape itself causes modification in aerodynamic loads, this interaction between the aerodynamic loads and elastic strength is called as aero-elastic effect.

A. WING TORSIONAL DIVERGENCE

1. Consider the wing at an incidence; the pressure distribution is such that the main load is located near the nose head of the Torsional axis which causes the wing to twist in nose-up sense.
2. Twist gives increment in incidence, which gives increment in lift force, which causes the wing to twist more and more about wing Torsional axis.
3. The tendency to twist is resisted by elastic force due to stiffness or strength of the wing.
4. As the speed increases an aerodynamic force increases rapidly, whereas the elastic stiffness is not affected by the lift and therefore an amount of twist increases with increase in speed.
5. A speed is reached at which an elastic resistance to the twist is only sufficient to counter balance the twisting moment due to lift.
6. This equilibrium is reached at the breaking point and the speed at which this equilibrium is reached is called as "WING TORSIONAL DIVERGENCE SPEED".
7. Any increase in the speed above this value will result in structural failure; this phenomenon is called as "WING TORSIONAL DIVERGENCE".

B. CONTROL SUURFACE REVERSAL/ AILERON REVERSAL

1. Consider a wing with aileron; an aileron is deflected downwards to give increment in the lift.
2. Downward deflection of aileron modifies the pressure distribution over the wing in such a way that the line of action of an aerodynamic force lies behind the wing Torsional axis and therefore wing is twisted in nose down sense.
3. As the wing is twisted in nose down dense, it causes reduction in the effective incidence and hence lift is reduced.

4. The tendency to the twist is resisted by elastic strength of the structure.
5. As the speed increases, for given aileron deflection, the wing gets twisted more and more in nose down sense, which decreases lift further and aileron effectiveness decreases.
6. The critical speed at which the loss of lift due to nose down twist is exactly equal to increment in lift due to aileron f=deflection is called as "AILERON REVERSAL SPEED".
7. Above this speed aileron is totally ineffective and downward deflection of aileron will result in rolling moment to the same wing OR it produces reverse rolling moment, this phenomenon is known as "CONTAOL SURFACE REVERSAL OR AILERN REVERSAL".

C. CONTROL SURFACE FLUTTER

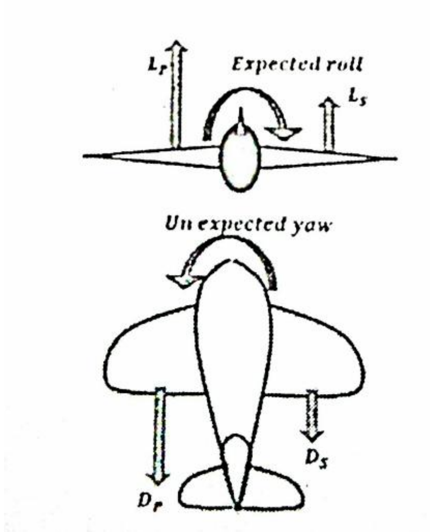
1. It is due to the interaction between inertia force of control surface and aerodynamic and elastic and elastic effects of the structure.
2. Consider an elevator hinged along a line which is ahead of the centre of gravity of an elevator.
3. Now in steady state and level condition, if the disturbance causes the tail to be displaced upwards, due to inertia force elevator is deflected downwards and gives increment in lift.
4. Increment in lift tries to bend the structure of fuselage, which causes the tail to be displaced further upwards.
5. However oscillations may be created developed under certain circumstances which are known as "ELEVATOR FLUTTER OR CONTROL SURFCE FLUUTTER".
6. Control surface flutter can be avoided by-
    a. Increasing the rigidity of the structure
    b. Mass balancing.

D. MASS BALANCING

1. The careful arrangement of mass distribution of the elevator in order to eliminate the inertia aero-elastic coupling which produces flutter is called as MASS BALANCING.
2. In case of mass balancing an auxiliary mass is fitted to the control surface ahead of the hinge line.
3. Large transport aircrafts or military aircrafts now uses the power controls which are less sensitive to the flutter due to high rigidity of the actuating cylinders.

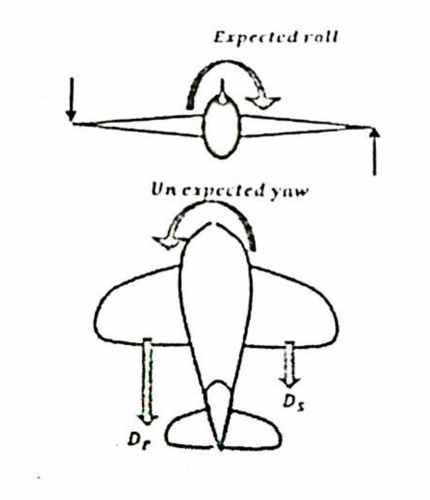
CROSS COUPLING EFFECTS

A. Yawing moment due to aileron deflection (Adverse aileron yaw)



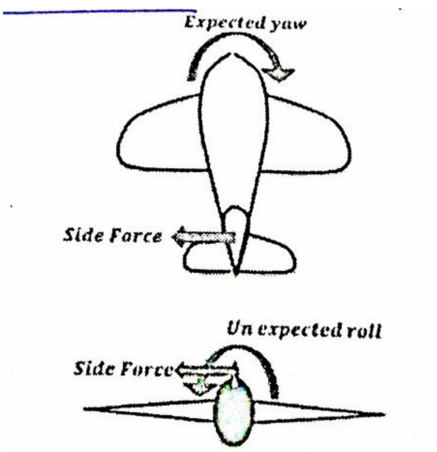
1. Ailerons are deflected differentially in order to create a rolling moment.
2. E.g., to roll the aircraft to the starboard side the aileron of starboard wing is deflected upwards and that of the port wing is deflected downward.
3. Due to this deflection it increases the life of the port wing and decreases the lift of starboard wing thereby giving the expected rolling moment.
4. But the lift does not increase alone; the increase in lift of port wing increases the drag also by a considerable amount due to increment in induced drag.
5. The drag of starboard wing is comparatively less than the drag of port wings, this difference in drag causes the aircraft yaw towards the port wing.
6. Since the yawing moment is in the opposite sense to the expected rolling moment the phenomenon is called as adverse aileron yaw.

B. Yawing moment due to rolling velocity



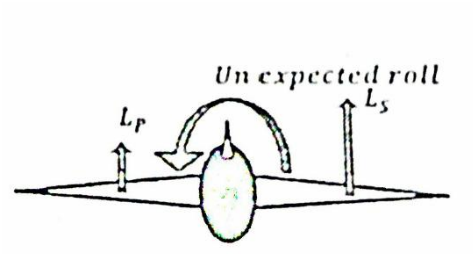
1. Yawing moment due to rolling velocity arises from tilting of the lift vectors.
2. E.g. when an airplane is rolled to the starboard side, the starboard wing experiences up-wash and lift vector is tilted forwards, thereby reducing the drag of the starboard wing.
3. At the same time port wing experiences down-wash due to which the lift vector is tilted backwards, thereby increasing the drag of the port wing.
4. This difference in drag causes unexpected yawing moment which is again in the opposite sense to the expected rolling moment.

C. Rolling moment due to rudder deflection



1. The rudder is deflected to the starboard in order to initiate the expected yawing moment towards the starboard.
2. But when the rudder is deflected to starboard, it creates a side force towards the port wing.
3. Since the line of action of this force is above the C.G. or longitudinal axis, it causes the aircraft to roll towards the port side.

D. Rolling moment due to yawing velocity



1. When an airplane is yawing to the port side, the starboard wing travels faster than port wing and therefore the lift of the starboard wing is greater than the lift of the port wing.
2. This difference in lift causes the aircraft to roll to the port side, which is in the same sense as that of the original yaw.