TORSION

$$\frac{\boldsymbol{T}}{\boldsymbol{J}} = \frac{\boldsymbol{\tau}}{\boldsymbol{r}} = \frac{\boldsymbol{G\theta}}{\boldsymbol{l}}$$

Where,

T = torque applied, J = Polar moment of inertia, τ =Shear stress at distance r, r = perpendicular distance

G = modulus of rigidity, $\frac{\theta}{l}$ = Angular twist per unit length, GJ = Torsional rigidity

1. **For solid circular cross-sections :**

   J = $I_{xx} + I_{yy}$
   = $\frac{\pi d^4}{64} + \frac{\pi d^4}{64}$
   = $\frac{\pi d^4}{32}$

   $\tau = \frac{T}{J}$r
   $\frac{d\theta}{dz} = \frac{T}{GJ}$

   Note:

   - For thick circular ring:

     J = $\frac{\pi}{32}[d_2^4 - d_1^4]$

   - For circular sections, τ is maximum at the outer most section.

2. **For hollow thin walled circular cross-sections :**
   J = $I_{xx} + I_{yy}$
   = πR^3t + πR^3t
   = 2πR^3t

   $\tau = \frac{T}{J}$r

3. **Thin walled open cross sections :**

   $$\frac{\boldsymbol{T}}{\boldsymbol{J}} = \frac{\boldsymbol{\tau}}{\boldsymbol{t}} = \frac{\boldsymbol{G\theta}}{\boldsymbol{l}}$$

   J = $\sum \frac{bt^3}{3}$ where b = length
   $\frac{d\theta}{dz} = \frac{T}{GJ}$
   $\tau_{max} = \frac{T}{J} t_{max}$
   Here, J is known as Torsional constant or St. Venant constant for thin walled open cross-section.

4. **Thin walled closed cross-sections :**

J = $\frac{4A^2}{\int \frac{ds}{t}}$

$$\frac{d\theta}{dz} = \frac{T}{GJ}$$
$$= \frac{T}{4A^2G}\oint\frac{ds}{t}$$
$$= \frac{q}{2AG}\oint\frac{ds}{t}$$

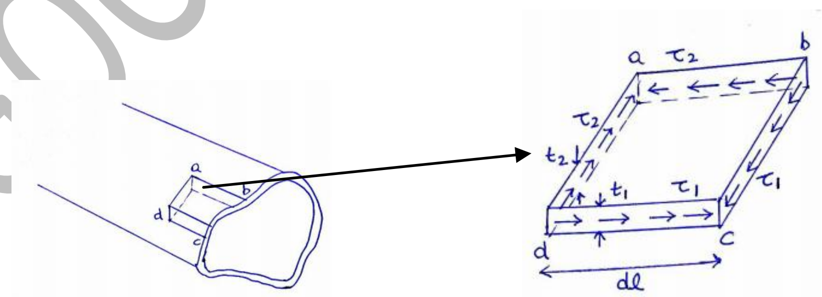
T = 2Aq, where q = τt (N/mm)

NOTE:

- For closed thin walled sections, maximum shear stress is inversely proportional to thickness.
- For thin walled open sections, maximum shear stress is directly proportional to thickness.

S.No.	***c/s***	***J***	$\boldsymbol{\tau}$	$\boldsymbol{\frac{d\theta}{dz}}$
1	Circular section (Solid)	$\frac{\pi d^4}{32}$	$\tau = \frac{T}{J}$r	$\frac{T}{GJ}$
2	Circular section (Hollow)	2πR^3t	$\tau = \frac{T}{J}$r	$\frac{T}{GJ}$
3	Thin-walled (Open)	$\frac{bt^3}{3}$	$\tau = \frac{T}{J}$t	$\frac{T}{GJ}$
4	Thin-walled (Closed)	$\frac{4A^2}{\int\frac{ds}{t}}$	q = τt, T = 2Aq	$\frac{d\theta}{dz} = \frac{T}{GJ} = \frac{T}{4A^2G}\oint\frac{ds}{t} = \frac{q}{2AG}\oint\frac{ds}{t}$

<u>TORSION OF THIN WALLED CLOSED TUBES (</u>**<u>BREDT – BATHO THOERY) –</u>**



1. Fig shows thin walled cylindrical tube which is subjected to pure Torsional moments T.
2. The tube ends and the c/s are free to warp out of their plane i.e. there are no longitudinal normal stresses.

3. Hoope stresses are zero (no tangential normal stresses).
4. The c/s thickness may vary along the circumference but it is constant along longitudinal axis.
5. The shear stress perpendicular to longitudinal axis are neglected..

Consider an element abcd of the tube wall. For a plate in pure shear, longitudinal shear stresses are constant in longitudinal direction. Considering equilibrium of element of element in Z direction,

$$\sum F_Z = 0$$

$$\tau_1 t_1 dl - \tau_2 t_2 dl = 0$$

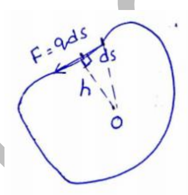
$$\tau_1 t_1 dl = \tau_2 t_2 dl$$

q = $\boldsymbol{\tau t}$, constant (shear flow)

$$\tau = \frac{q}{t}$$

Hence the constant value (τt) is called the shear flow. This is also known as Bredt–Batho shear. The units of shear flow are N/mm, also defined as force per unit length.

Take the moment of shear flow of the c/s about any interior pt. O, in the c/s



T = $\int F.h = \int q.hds$, where $\int hds = 2A$ ($\frac{1}{2} \times h \times ds = dA$)

T = 2Aq, which is known as Bredt–Batho formula

Note: A = cell area

SHAFTS IN PARALLEL

1. **Torque taken will not be same i.e.**

$$T_1 \neq T_2$$
$$and\ T = T_1 + T_2$$
$$= 2A_1q_1 + 2A_2q_2$$

2. **Angular twist remains constant**

$$(\frac{d\theta}{dz})_1 = (\frac{d\theta}{dz})_2$$

SHAFTS IN SERIES

1. **Torque remains constant**

$$T_1 = T_2$$

$$i.e. q = q_2$$

2. ANGULAR TWIST REMAINS CONSTANT

$$\frac{d\theta}{dz} = (\frac{d\theta}{dz})_1 + (\frac{d\theta}{dz})_2$$

STRAIN ENERGY IN BENDING

$$U = \frac{1}{2} T \times \theta$$

Now,

$$\frac{T}{J} = \frac{\tau}{r} = \frac{G\theta}{l}$$

$$U = \frac{1}{2} \frac{T^2 l}{GJ}$$

$$U = \int_0^l \frac{T^2 dx}{2GJ}$$

COMBINED BENDING AND TORSION

1. Shear stresses due to torsion
2. Normal stresses due to bending
3. Shear stresses due to bending are neglected.

T = P.r

M = P.x

$\tau_{max} = \frac{T}{J} t_{max} = \frac{16T}{\pi d^3}$

$\sigma_x = \sigma_{max} = \frac{M}{I} y = \frac{32M}{\pi d^3}$

$\sigma_y = 0$

$$\sigma = \left(\frac{\sigma_{xx} + \sigma_{yy}}{2}\right) + \sqrt{\left(\frac{\sigma_{xx} - \sigma_{yy}}{2}\right)^2 + \sigma_{xy}{}^2}$$

$$\sigma = \left(\frac{\sigma_{xx}}{2}\right) + \sqrt{\left(\frac{\sigma_{xx}}{2}\right)^2 + \sigma_{xy}{}^2}$$

$$\sigma_{max} = \sigma_I = \frac{16M}{\pi d^3} + \sqrt{\left(\frac{16M}{\pi d^3}\right)^2 + \left(\frac{16T}{\pi d^3}\right)^2}$$

$$\boldsymbol{\sigma_{max} = \sigma_I = \frac{16}{\pi d^3} [\, M + \sqrt{(M)^2 + (T)^2}\,]}$$

$$\boldsymbol{\sigma_{min} = \sigma_I = \frac{16}{\pi d^3} [\, M - \sqrt{(M)^2 + (T)^2}\,]}$$

$$\boldsymbol{\tau_{max} = \frac{16}{\pi d^3} [\sqrt{(M)^2 + (T)^2}\,]}$$