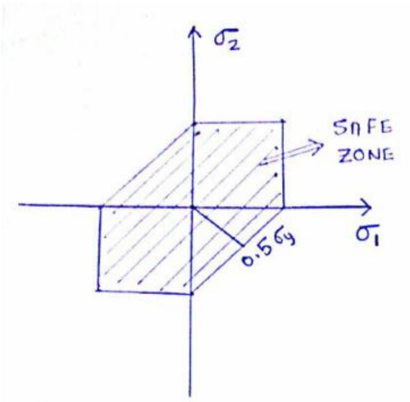
THEORY OF FAILURE

1. Maximum Shear stress theory (Tresca Theory)

- Ductile Materials
- Yielding would occur when the maximum shear stress exceeds the shear stress at the tensile yield point.
- Graph is a hexagon.

$$\textbf{Max } \left|\frac{\sigma_1-\sigma_2}{2}, \frac{\sigma_2-\sigma_3}{2}, \frac{\sigma_3-\sigma_1}{2}\right| < \tau_y$$

$$\frac{\tau_y}{\sigma_y} = 0.5$$



2. Distortion energy theory (von-Mises yield criteria/ Maximum shear strain energy theory)

- Same as octahedral shear stress theory
- Ductile Materials
- Yielding would occur when the distortion energy exceeds the value of distortion energy at the yield point in a simple tension test.

$$\left(\frac{1+\vartheta}{6E}\right)\left[(\sigma_1-\sigma_2)^2+(\sigma_2-\sigma_3)^2+(\sigma_3-\sigma_1)^2\right] \le u_{max}$$

In tension,

$$u_{max} = \frac{2(1+\vartheta)\sigma_{y^2}}{6E} = \frac{\sigma_{y^2}}{6G}$$

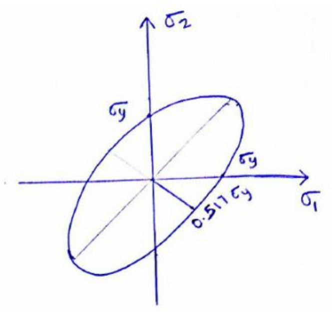
$$(\sigma_1-\sigma_2)^2+(\sigma_2-\sigma_3)^2+(\sigma_3-\sigma_1)^2 \le 2\sigma_{y^2}$$

In Torsion,

$$\sigma_1 = \tau, \sigma_2 = 0, \sigma_3 = -\tau$$

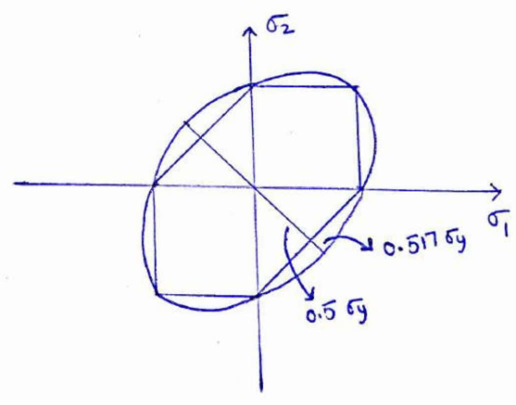
$$\frac{6(1+u)\tau_{y^2}}{6E} = u_{max}$$

$$\frac{\tau_y}{\sigma_y} = \frac{1}{\sqrt{3}}$$



NOTE: Comparison between Tresca and Von-Mises theory

- Tresca is more conservative theory i.e. more cost and heavier design.

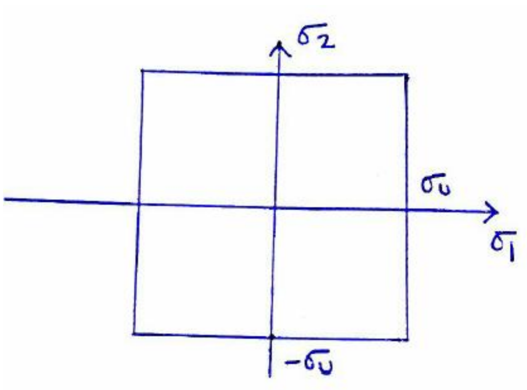


3. Max principle stress theory (Rankine theory)

- Brittle materials
- If one of the principles stress σ_1 and σ_2 and exceeds the ultimate stress, failure would occur.

$$\textbf{Max } |\boldsymbol{\sigma_1, \sigma_2, \sigma_3}\ | \leq \boldsymbol{\sigma_u}$$

$$\boldsymbol{\tau_u = \sigma_u}$$



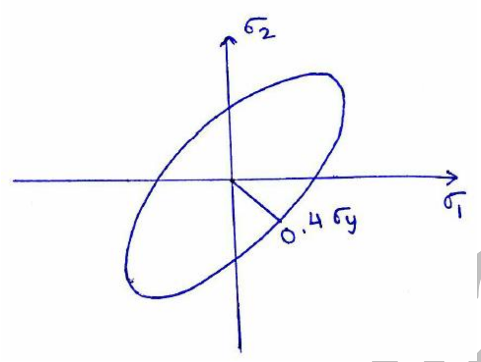
- Graph is a square.
- Drawback – Take tensile and compressive strength as same.
- Mohr's Theory is used to overcome this drawback, i.e. different strength in tensile and compressive.

4. Maximum strain energy theory (Haigh's theory)

- Used for Brittle materials.

$$(\sigma_1 - \sigma_2 + \sigma_3)^2 - 2(1+\vartheta)(\sigma_1\sigma_2 + \sigma_2\sigma_3 + \sigma_3\sigma_1) < \sigma_y^2$$

$$\frac{\tau_y}{\sigma_y} = 0.4 = \frac{1}{2(1+\vartheta)}$$



5. Max principal strain theory (St. Venant's theory)

- Used for Brittle materials.
- Graph is a rhombus.

$$\frac{1}{E}(\sigma_1 - \vartheta(\sigma_2 + \sigma_3)) < \varepsilon_{max}$$

In tension, $\sigma_1 - \vartheta(\sigma_2 + \sigma_3) < \sigma_y$

In torsion, $\sigma_1 = \tau, \sigma_2 = 0, \sigma_3 = -\tau$

$$\varepsilon_{max} = \frac{\tau}{E} + \frac{\vartheta\tau}{E} = \frac{\tau}{E}(1+\vartheta)$$

$$\tau_y = \frac{E}{(1+\vartheta)} E_{max}$$

$$\frac{\tau_y}{\sigma_y} = \frac{1}{1+\vartheta} = 0.8 \text{ at } \vartheta = 0.25$$