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AERODYNAMICS & SPACE DYNAMICS

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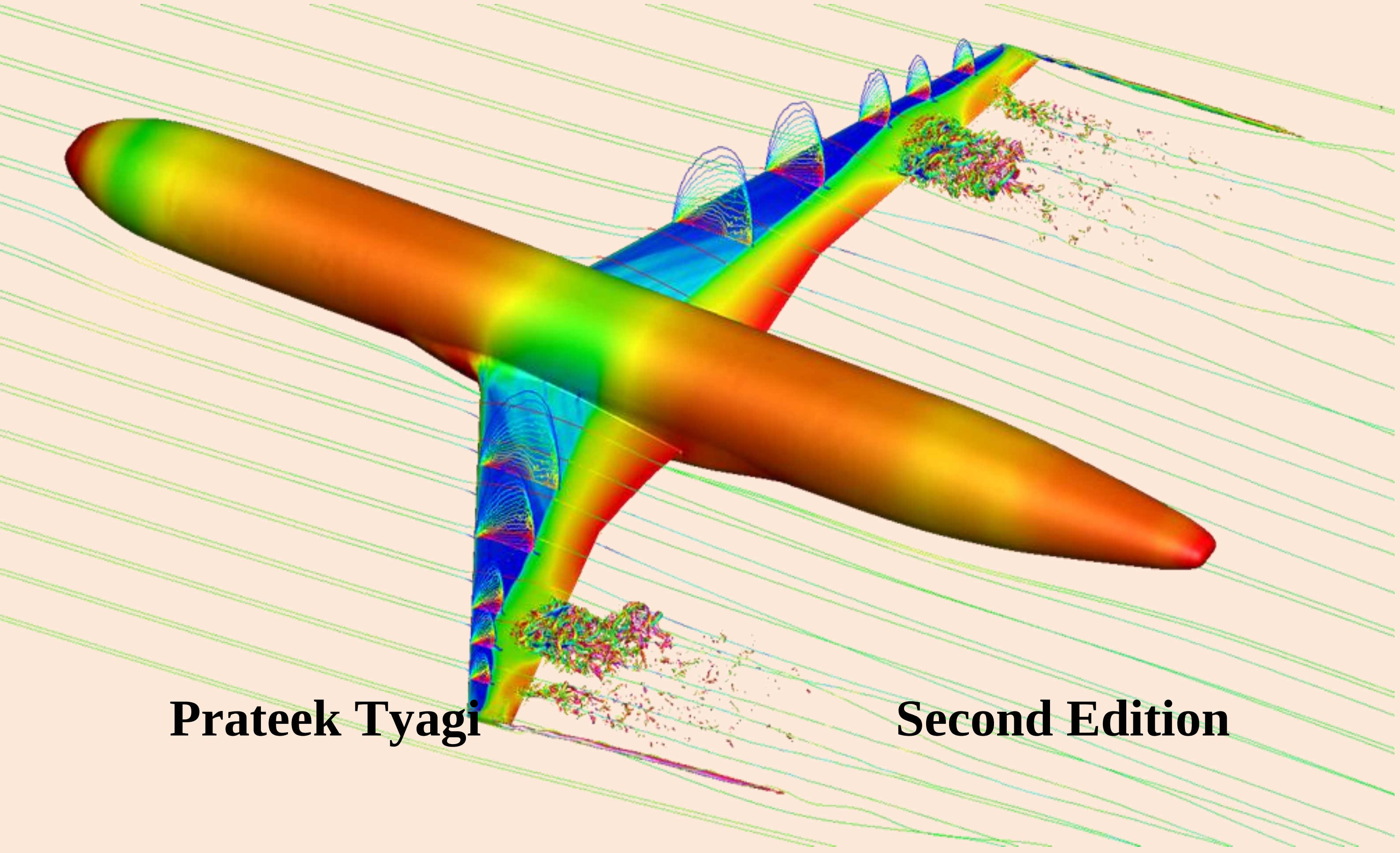


Prateek Tyagi

Second Edition

GATE 2019

ORGANIZING INSTITUTE :

IIT MADRAS

AEROSPACE ENGINEERING

Aerodynamics Syllabus

Core Topics:

Basic Fluid Mechanics: Conservation laws: Mass, momentum (Integral and differential form); Potential flow theory: sources, sinks, doublets, line vortex and their superposition; Viscosity, Reynold's number.

Airfoils and wings: Airfoil nomenclature; Aerodynamic coefficients: lift, drag and moment; Kutta-Joukoswki theorem; Thin airfoil theory, Kutta condition, starting vortex; Finite wing theory: Induced drag, Prandtl lifting line theory; Critical and drag divergence Mach number.

Compressible Flows: Basic concepts of compressibility, Conservation equations; One dimensional compressible flows, Fanno flow, Rayleigh flow; Isentropic flows, normal and oblique shocks, Prandtl-Meyer flow; Flow through nozzles and diffusers.

Special Topics:

Elementary ideas of viscous flows including boundary layers; Wind Tunnel Testing:
Measurement and visualization techniques.

Aerodynamics Year Wise Analysis

Year	No of Questions	Topics (1 marks + 2 marks)	Total Marks
2018	1M : 6 2M : 6	• Basic Fluid mechanics and conservation eq. (1 + 1) • Ideal potential flow (1 + 0) • Thin airfoil theory (1 + 0) • Low Speed Aerodynamics (0 + 1) • High lift devices and NACA airfoils (1 + 0) • Gas Dynamics (2 + 4)	18
2017	1M : 5 2M : 7	• Basic Fluid mechanics and conservation eq. (2 + 4) • Low Speed Aerodynamics (1 + 0) • Finite wing theory (0 + 1) • Gas Dynamics (2 + 2)	19
2016	1M : 7 2M : 4	• Basic Fluid mechanics and conservation eq. (1 + 0) • Ideal potential flow (1 + 1) • Thin airfoil theory (0 + 1) • Low Speed Aerodynamics (1 + 0) • Finite wing theory (0 + 1) • High lift devices and NACA airfoils (1 + 0) • High Speed Airfoils (1 + 0) • Gas Dynamics (2 + 1)	15
2015	1M : 7 2M : 6	• Basic Fluid mechanics and conservation eq. (1 + 1) • Ideal potential flow (0 + 1) • Thin airfoil theory (0 + 2) • Low Speed Aerodynamics (2 + 0) • Finite wing theory (1 + 0) • High lift devices and NACA airfoils (2 + 0) • Gas Dynamics (1 + 2)	19
2014	1M : 5 2M : 8	• Basic Fluid mechanics and conservation eq. (1 + 3) • Ideal potential flow (1 + 0)	21

		• Low Speed Aerodynamics (0 + 1) • Finite wing theory (0 + 1) • High lift devices and NACA airfoils (1 + 0) • High Speed Airfoils (0 + 1) • Gas Dynamics (2 + 2)	
2013	1M : 5 2M : 7	• Basic Fluid mechanics and conservation eq. (1 + 1) • Ideal potential flow (1 + 1) • Thin airfoil theory (0 + 2) • Finite wing theory (0 + 1) • High Speed Airfoils (2 + 0) • Gas Dynamics (1 + 2)	19
2012	1M:6 2M:5	• Basic Fluid mechanics and conservation eq. (1 + 2) • Ideal potential flow (1 + 0) • Thin airfoil theory (0 + 1) • Low Speed Aerodynamics (1 + 0) • Finite wing theory (1 + 0) • High Speed Airfoils (1 + 0) • Gas Dynamics (1 + 2)	16
2011	1M:4 2M:9	• Basic Fluid mechanics and conservation eq. (0 + 2) • Ideal potential flow (1 + 0) • Low Speed Aerodynamics (1 + 0) • Finite wing theory (0 + 4) • High lift devices and NACA airfoils (2 + 0) • Gas Dynamics (0 + 3)	22
2010	1M:5 2M:7	• Basic Fluid mechanics and conservation eq. (3 + 1) • Ideal potential flow (1 + 2) • Thin airfoil theory (0 + 1) • Finite wing theory (0 +1) • Gas Dynamics (1 + 2)	19
2009	1M:4 2M:6	• Basic Fluid mechanics and conservation eq. (0 + 2) • Ideal potential flow (1 + 1) • Thin airfoil theory (1 + 0) • Low Speed Aerodynamics (0 + 1) • Finite wing theory (0 + 1) • High lift devices and NACA airfoils (1 + 0)	16

		• Gas Dynamics (1 + 1)	
2008 (85 questions)	1M:4 2M:13	• Basic Fluid mechanics and conservation eq. (0 + 1) • Ideal potential flow (1 + 1) • Thin airfoil theory (0 + 2) • Finite wing theory (0 + 3) • High lift devices and NACA airfoils (2 + 0) • High Speed Airfoils (0 + 3) • Gas Dynamics (1 + 3)	30 (Total 150 marks)
2007 (85 questions)	1M:4 2M:8	• Basic Fluid mechanics and conservation eq. (0 + 3) • Ideal potential flow (0 + 2) • Thin airfoil theory (1 + 0) • Finite wing theory (0 + 2) • High Speed Airfoils (1 + 0) • Gas Dynamics (2 + 1)	20 (Total 150 marks)

TABLE OF CONTENT

- GATE and Additional questions
- Exercise at the end of each Chapter
- Previous year GATE questions solved

Chapter 2

GENERAL THIN AIRFOIL THEORY

2.1 ASSUMPTIONS

1. The airfoil is thin, so that its shape is effectively that of its camber line and also that the camber line shape deviates very slightly from the chord line.
2. Theory is resisted to low angle of attack.
3. The flow direction is always tangential to the surface of the airfoil.

The camber line is replaced by a line of variable vorticity so that the total circulation about the chord is sum of vortex element.

$$\mathbf{r} = \oint \boldsymbol{\Upsilon . ds}$$

where, ϒ is the distribution of vorticity over the element of camber line ds and circulation is taken positive in the anti-clockwise direction.

NOTE:

$$\mathrm{r} = \oint \overline{\mathrm{V}}.\overline{\mathrm{ds}} = \oiint (\nabla \times \overline{\mathrm{V}}).\overline{\mathrm{ds}} = \oiint \overline{\xi}.\overline{\mathrm{ds}}$$

$$\mathrm{r} = \oiint \overline{\xi}.\overline{\mathrm{ds}} = \oint \Upsilon \, \mathrm{ds} = \int_0^c \Upsilon \; \mathrm{ds}$$

$$\frac{\partial \mathbf{r}}{\partial \mathbf{S}} = \overline{\boldsymbol{\xi}}$$

i.e. change of circulation per unit area is vorticity.

Basic assumptions of theory permit the variation of vorticity along the camber line assumed to be same as the variation along the OX axis.

Hence for unit span of this section, the lift is given by

$$\text{Lift} = \rho U \, \mathrm{r} \qquad \text{N / unit span}$$

$$L = \rho U \oint \Upsilon . ds = \rho U \int_0^c \Upsilon \; dx \text{ --------------------- (1)}$$

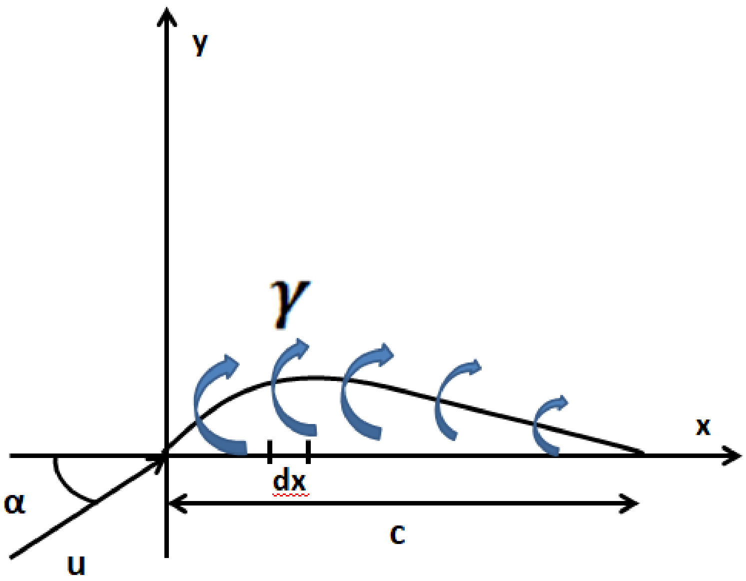
The net lift acting on the body can be written as, L = $\int_0^c (P_L - P_U)(dx \times 1)$

L = $\int_0^c \Delta P dx$-------------------- (2)

Comparing equation 1 and 2, we get,

$$\rho U \int_0^c \Upsilon \, dx = \int_0^c \Delta P dx$$

$$\boldsymbol{\Delta P = \rho U \Upsilon}$$



$$\Delta L = \Delta P dx cos\alpha$$

At small AOA cosα = 1

$$\Delta L = \Delta P dx$$

$$\mathbf{L = \int_0^c \Delta P dx = \rho U \int_0^c \Upsilon \, dx}$$

Moment about Leading edge

$M_{L.E.} = -\int_0^c \Delta P dx \times x$ (Nose down pitching moment is negative)

$$\boldsymbol{M_{L.E.} = -\rho U \int_0^c \Upsilon \; dx \times x}$$

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$$\Upsilon = (u + u_1) - (u + u_2)$$

$$\boldsymbol{\Upsilon = (V_1 - V_2)}$$

NOTE:

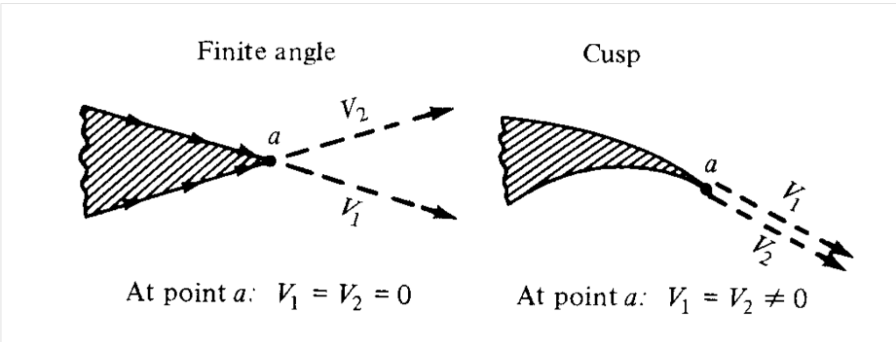
1. **Angled Trailing edge :**
   Since two streamlines cannot each other and it is impossible to have two velocities in two different directions at single point.
   Therefore, Angled Trailing edge must be a stagnation point.
   i.e. $V_1 = V_2 = 0$

hence,

$$\Upsilon = (V_1 - V_2) = 0$$

$$\boldsymbol{\Upsilon_{T.E.} = 0}$$

2. **Cusped trailing edge :**



$$P_\infty + \frac{1}{2}\rho U^2 = P_a + \frac{1}{2}\rho (V_1)^2$$

$$P_\infty + \frac{1}{2}\rho U^2 = P_a + \frac{1}{2}\rho (V_2)^2$$

i.e. $V_1 = V_2 \neq 0$

Hence,

$$\Upsilon = (V_1 - V_2) = 0$$

$$\boldsymbol{\Upsilon_{T.E.} = 0}$$

The above condition expressed in terms of the strength of the vortex sheet is known **as KUTTA CONDITION.**

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Aerodynamic co-efficients for a flat plate airfoil

(1). Lift coefficient

$$L = \int_0^c \Delta P dx = \rho U \int_0^c \Upsilon \, dx$$

$$\gamma = \frac{2U\alpha(1+cos\theta)}{sin\theta}$$

$$x = \frac{c}{2}(1 - cos\theta),\ dx = \frac{c}{2} sin\theta$$

$$L = \int_0^\pi \rho U \, \frac{2U\alpha(1+cos\theta)}{sin\theta} \, \frac{c}{2} sin\theta d\theta$$

$$L = \rho u^2 \alpha c \int_0^\pi (1 + \, cos\theta) d\theta$$

$$\frac{1}{2}\rho u^2 (c \times 1) c_L = \rho u^2 \alpha c \, \times \, \pi$$

$$\boldsymbol{c_L = 2\,\pi\alpha}$$

(2). Pitching Moment at Leading edge

$$M_{L.E.} = -\rho U \int_0^c \Upsilon \, xdx$$

$$\frac{1}{2}\rho u^2 c.c.c_{M\ L.E.} = -\rho U \int_0^c \Upsilon \, xdx$$

Substituting value of Υ in above expression and integrating, we get

$$\boldsymbol{c_{M\ L.E.} = \frac{-\pi\alpha}{2}}$$

NOTE:

(1) $\boldsymbol{\frac{c_{M\ L.E.}}{c_L} = \frac{-1}{4}}$

$$\boldsymbol{c_{M\ c/4} = c_{M\ L.E.} + \frac{c_L}{4}}$$

$$\boldsymbol{c_{M\ c/4} = 0}$$

Center of pressure as the point about which the moment are zero. For symmetric airfoil the center of pressure is at the quarter-chord point. Also, Point on an airfoil where moments are independent of angle of attack is called the aerodynamic center. The moment about quarter chord point is zero for all values of α. *Hence for symmetric airfoil, the quarter chord point is both the center of pressure and the aerodynamics center.*

(2) $M_{T.E.} = \rho U \int_0^c \Upsilon \ (c - x)dx$

Substituting value of ϒ in above expression and integrating, we get

$$c_{M\,T.E.} = \frac{3\pi\alpha}{2}$$

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2.7 KUTTA – ZOUKOWSKI'S TRANSFORMATION

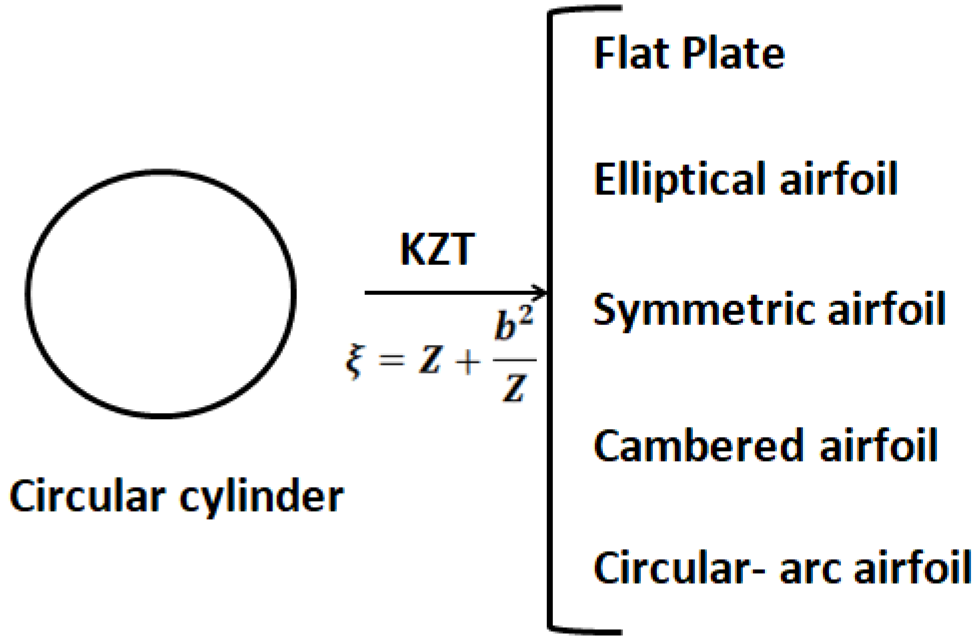
- Of all the transformation conformal in nature, the simplest transformation to produce conformal shape is KZT.

$$\xi = Z + \frac{b^2}{Z}$$

Where, b is constant

- The transformation can be applied to all the other streamlines for producing corresponding streamlines in ξ – plane flowing around the airfoil

- It transforms cylinder from the Z – plane to a special set of airfoils in ξ – plane. Such that the airfoils are known as Joukowsky's airfoils.



2.8 LIFT GENERATED BY ZOUKOWSKI'S AIRFOILS

$$C_L = 2\pi(1+e)sin(\alpha+\beta)$$

maximum thickness, $t_{max} = 3\sqrt{3}be$

chord length, $c = 4b$

Maximum camber, $c_{max} = 2b\beta$

$\frac{c_{max}}{c} = \frac{\beta}{2}$

where,

α → angle of attack of the airfoil
β → camber effect
e → eccentricity

Note: For flat plate airfoil, e = 0, $\beta = 0$ (no camber)

Hence,

$$C_L = 2\pi sin\alpha$$

For small α,

$$C_L = 2\pi\alpha$$

Q. Calculate the theoretical lift coefficient of a Zoukowski's airfoil t/c = 0.2 and 2% camber set at 4^0 incidence in a 2D irrotational flow.

Ans: 0.7946

Given: $\alpha = 4^0$

$$\frac{t}{c} = 0.2 = 1.3e$$

$$\rightarrow e = 0.154$$

$$\frac{c_{max}}{c} = \frac{\beta}{2} = 0.02$$

$$\rightarrow \beta = 0.04 \times \frac{180}{\pi} = 2.292$$

$$C_L = 2\pi(1 + e)\sin(\alpha + \beta)$$

$$\boldsymbol{C_L = 0.7946}$$

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GATE and Additional objective questions

Q1. The rate of change of moment coefficient with respect to the angle of attack, $\frac{dC_m}{d\alpha}$ at half chord point of a thin airfoil, as per approximations frm the thin airfoil theory is

(a) π/4 radian^{-1} (b) π/2 radian^{-1} (c) πradian^{-1} (d) 2π radian^{-1} **[GATE 2016]**

Ans: (b) π/2 radian^{-1}

Pitching Moment at half chord point of a thin airfoil

$$M_{half-point} = \rho U \int_0^c \Upsilon\left(\frac{c}{2} - x\right)dx$$

$$\gamma = \frac{2U\alpha(1+cos\theta)}{sin\theta}$$

$$\frac{1}{2}\rho U^2 c^2 C_{m,half-point} = \rho U \int_0^c \left(\frac{2U\alpha(1+cos\theta)}{sin\theta}\right)\left(\frac{c}{2} - x\right)dx$$

$$x = \frac{c}{2}(1 - cos\theta),\ dx = \frac{c}{2}sin\theta$$

$$\frac{1}{2}\rho U^2 c^2 C_{m,half-point} = \rho U \int_0^\pi \left(\frac{2U\alpha(1+cos\theta)}{sin\theta}\right)\left(\frac{c}{2}cos\theta\right)\frac{c}{2}sin\theta d\theta$$

$$C_{m,half-point} = \frac{\pi\alpha}{2}$$

$$\boldsymbol{\frac{dC_m}{d\alpha} = \frac{\pi}{2}}$$

Q2. Consider a symmetric airfoil at an angle of attack of 4^0. Using thin airfoil theory, the magnitude of the moment coefficient about the leading edge is **[GATE 2008]**

(a) 2π (b) π (c) $\frac{\pi^2}{60}$ (d) $\frac{\pi^2}{90}$

Ans: (d) $\boldsymbol{\frac{\pi^2}{90}}$

$$c_{M\ L.E.} = \frac{-\pi\alpha}{2}$$

Where, α is in radians

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Chapter 7

LINEARIZED SUPERSONIC FLOW

7.1 INTRODUCTION

The linearized perturbation velocity potential equation is given by

$$(1 - M_\infty^2)\frac{\partial^2 \hat{\varphi}}{\partial x^2} + \frac{\partial^2 \hat{\varphi}}{\partial y^2} = 0$$

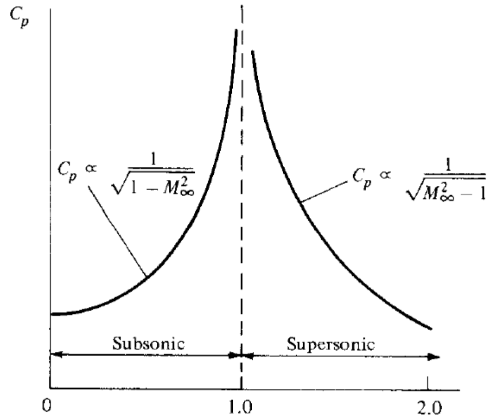
The above equation holds for both subsonic and supersonic flows.

7.2 LINEARIZED SUPERSONIC PRESSURE COEFFICIENT FORMULA

$$C_P = \frac{2\theta}{\sqrt{M_\infty^2 - 1}}$$

It is the linearized supersonic pressure coefficient, and it states that C_P is directly proportional to the local surface inclination with respect to the freestream. It holds for any slender two – dimensional body where Ө is small.

Note that Ө is positive when measured above the horizontal, and negative when measured below the horizontal.

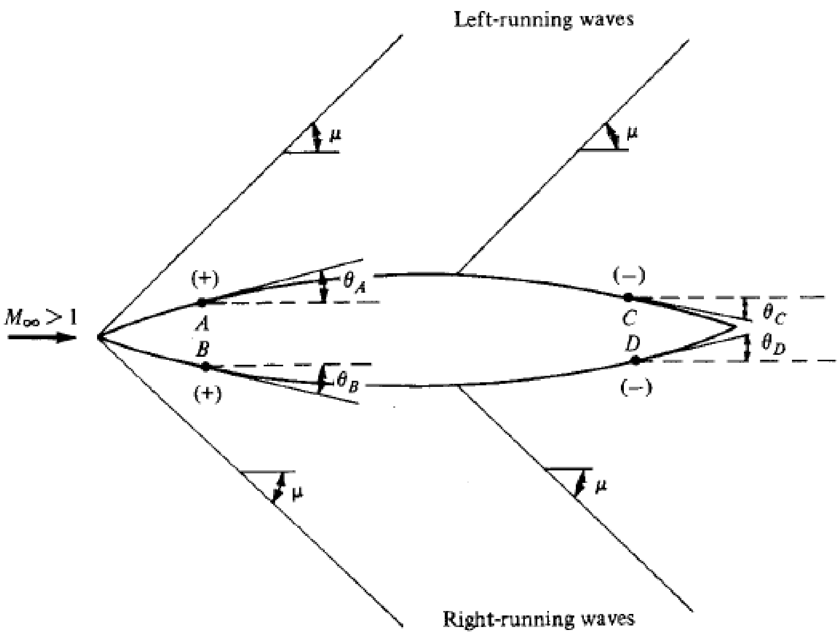


- Note that $C_P \propto (M_\infty^2 - 1)^{-1/2}$; hence, for supersonic flow, C_P decreases as M_∞ increases.
- This is in contrast with subsonic flow, where $C_P \propto (1 - M_\infty^2)^{-1/2}$; hence for subsonic flow, C_P increases as M_∞ increases.

$$C_{P_{compressible(subsonic)}} = \frac{C_{P_{incompressible}}}{\sqrt{\mathbf{1 - M_\infty^2}}}$$

- Both results predict $C_P \rightarrow \infty \; as \; M_\infty \rightarrow 1$ from either side.

7.3 APPLICATION TO SUPERSONIC AIRFOILS



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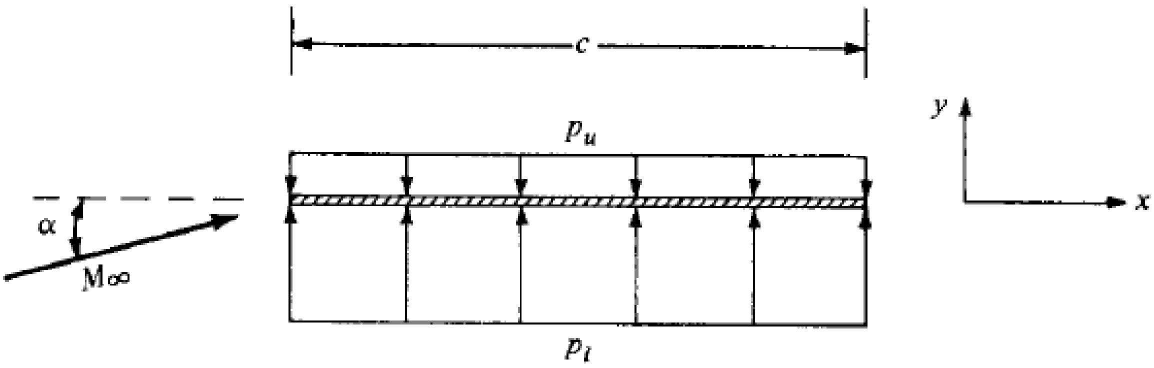
7.4 A FLAT PLATE AT ANGLE OF ATTACK IN A SUPERSONIC FLOW



Looking at this picture, the lower surface of the plate is a compression surface inclined at the angle α into the freestream, given by

$$\boldsymbol{C_{P,l} = \frac{2\alpha}{\sqrt{M_\infty^2 - 1}}}$$

Since the surface inclination angle is constant along the entire lower surface, $C_{P,l}$ is a constant value over the lower surface.

Similarly, the top surface is an expansion surface inclined at the abgle α away frim the freestream, given by

$$\boldsymbol{C_{P,u} = -\frac{2\alpha}{\sqrt{M_\infty^2 - 1}}}$$

$C_{P,u}$ is a constant value over the upper surface.

The normal force coefficient **C**$_n$ be obtained from

$$c_n = \frac{1}{c}\int_0^c (C_{p,l} - C_{p,u})dx$$

$$c_n = \frac{\boldsymbol{4\alpha}}{\sqrt{\boldsymbol{M_\infty^2 - 1}}}\frac{1}{c}\int_0^c dx = \frac{\boldsymbol{4\alpha}}{\sqrt{\boldsymbol{M_\infty^2 - 1}}}$$

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Q1. Consider a thin flat plate airfoil at a small angle α to an oncoming supersonic stream of air. Assuming the flow to be Inviscid, $\frac{C_d}{C_l^2}$ **is** **[GATE 2013]**

(a) zero (b) independent of α

(c) proportional to α (d) proportional to α^2

Ans: (b) independent of α

We know,

$$C_l = \frac{4\alpha}{\sqrt{M_\infty^2 - 1}}$$

$$C_d = \frac{4\alpha^2}{\sqrt{M_\infty^2 - 1}}$$

$$\frac{C_d}{C_l^2} = \frac{\sqrt{M_\infty^2 - 1}}{4}$$

Hence, it is independent of α

Q2. Using linearized theory, calculate the lift and drag coefficients for a flat plate at a 5^0 angle of attack in a Mach 3 flow.

Ans: $\boldsymbol{\alpha = 5^0 = 0.087\ rad}$

$$C_l = \frac{4\alpha}{\sqrt{M_\infty^2 - 1}} = \frac{4(0.087)}{\sqrt{3^2 - 1}} = 0.123$$

$$C_d = \frac{4\alpha^2}{\sqrt{M_\infty^2 - 1}} = \frac{4(0.087^2)}{\sqrt{3^2 - 1}} = 0.011$$

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