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FLIGHT MECHANICS

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Third Edition

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GATE 2020

ORGANIZING INSTITUTE :

IIT DELHI

AEROSPACE ENGINEERING

Flight Mechanics Syllabus

Core Topics:

Basics: Atmosphere: Properties, standard atmosphere. Classification of aircraft. Airplane (fixed wing aircraft) configuration and various parts.

Airplane performance: Pressure altitude; equivalent, calibrated, indicated air speeds; Primary flight instruments: Altimeter, ASI, VSI, Turn-bank indicator. Drag polar; takeoff and landing; steady climb & descent, absolute and service ceiling; cruise, cruise climb, endurance or loiter; load factor, turning flight, V-n diagram; Winds: head, tail & cross winds.

Static stability: Angle of attack, sideslip; roll, pitch & yaw controls; longitudinal stick fixed & free stability, horizontal tail position and size; directional stability, vertical tail position and size.

Special Topics:

Dynamic stability: Euler angles; Equations of motion; aerodynamic forces and moments, stability & control derivatives; decoupling of longitudinal and lateral-directional dynamics; longitudinal modes; lateral-directional modes.

Flight Mechanics Year Wise Analysis

Year	No of Questions	Topics (1 marks + 2 marks)	Total Marks
2019	1M : 5 2M : 5	• Steady, state and level flight (2 + 1) • Dynamic Stability (1 + 1) • Aircraft Speeds (1 + 0) • Gliding (1 + 0) • Static Stability (0 + 2) • Endurance (0 +1)	15
2018	1M : 4 2M : 7	• Atmosphere (0 + 1) • Steady state & Level Flight (1 + 4) • Accelerated Flight Performance (3 + 0) • Static Stability (0 + 1) • Equations of motion(0+ 1)	18
2017	1M : 3 2M : 6	• Steady state & Level Flight (0 + 2) • Accelerated Flight Performance (1 + 1) • Static Stability (2 + 2) • Dynamic Stability (0 + 1)	15
2016	1M : 5 2M : 5	• Steady state & Level Flight (3 + 2) • Accelerated Flight Performance (1 + 0) • Static Stability (0 + 1) • Dynamic Stability (1 + 1) • Equations of motion(0+ 1)	15
2015	1M : 5 2M : 4	• Atmosphere (1 + 0) • Steady state & Level Flight (2 + 2) • Accelerated Flight Performance (1 + 0) • Static Stability (0 + 1) • Dynamic Stability (1 + 0) • Equations of motion(0+ 1)	13
2014	1M : 4 2M : 5	• Steady state & Level Flight (3 + 2) • Accelerated Flight Performance (0 + 2) • Static Stability (0 + 1)	14

		• Dynamic Stability (1 + 0)	
2013	1M : 5 2M : 7	• Steady state & Level Flight (2 + 3) • Accelerated Flight Performance (0 + 1) • Static Stability (2 + 3) • Dynamic Stability (1 + 0)	19
2012	1M:6 2M:6	• Steady state & Level Flight (1 + 2) • Accelerated Flight Performance (1 + 2) • Static Stability (2 + 2) • Dynamic Stability (1 + 0) • Equations of motion(1 +0)	18
2011	1M:3 2M:5	• Steady state & Level Flight (2 + 2) • Accelerated Flight Performance (0 + 2) • Static Stability (1 + 0) • Dynamic Stability (0 + 1)	13
2010	1M:4 2M:5	• Steady state & Level Flight (2 + 0) • Accelerated Flight Performance (2 + 1) • Static Stability (0 + 2) • Dynamic Stability (0 + 2)	14
2009	1M:4 2M:7	• Steady state & Level Flight (2 + 1) • Static Stability (2 + 2) • Dynamic Stability (0 + 2) • Equations of motion(0 +2)	18
2008 (85 questions)	1M:4 2M:13	• Atmosphere (0 + 1) • Steady state & Level Flight (3 + 4) • Accelerated Flight Performance (1 + 1) • Static Stability (0 + 5) • Dynamic Stability (0 + 1) • Equations of motion(0 +1)	30 (Total 150 marks)
2007 (85 questions)	1M:5 2M:12	• Atmosphere (0 + 0) • Steady state & Level Flight (2 + 4) • Accelerated Flight Performance (0 + 0) • Static Stability (3 + 7) • Dynamic Stability (0 + 1) • Equations of motion(0 +0)	29 (Total 150 marks)

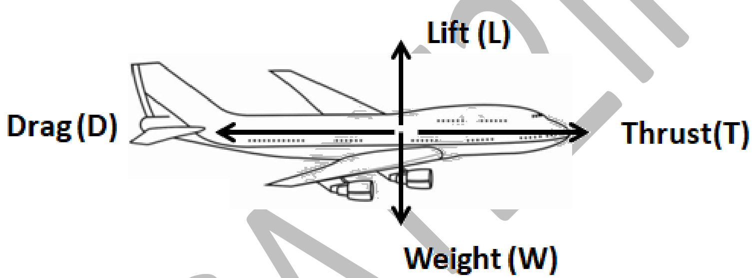
TABLE OF CONTENT

Chapter 2

AIRPLANE PERFORMANCE IN STEADY & LEVEL FLIGHT

For equilibrium, T = D and L=W

Since, $L = \frac{1}{2}\rho V^2 S C_L$



Therefore, $V = \sqrt{\frac{2W}{\rho S C_L}}$

i.e. $V \propto \sqrt{\frac{1}{C_L}}$

2.1 STALLING SPEED (V_{stall})

It is the minimum speed with which an airplane can fly without losing its control, called as the stalling speed, occurs at C_{Lmax}.

$$V_{stall} = \sqrt{\frac{2W}{\rho S C_{Lmax}}}$$

2.2 WING LOADING (w)

It is the ratio of the weight of an airplane to its wing area.

$$w = \frac{W}{S}$$

2.3 DRAG

The drag polar equation is given as

$$C_D = a + bC_L^2 \text{ or } C_D = C_{D0} + kC_L^2$$

Where C_D is Total drag coefficient.

a or C_{D0} is zero lift drag coefficient/ Profile drag coefficient/ parasite drag coefficient

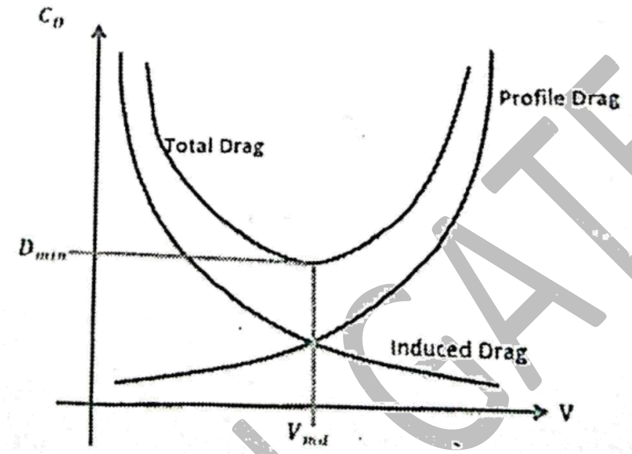
bC_L^2 or kC_L^2 is induced drag coefficient/ Lift dependent drag coefficient/ Trailing vortex drag coefficient, Where b = k = 1/πeAR

Now, $D = \frac{1}{2}\rho V^2 S C_D$

$D = \frac{1}{2}\rho V^2 S\,(a + bC_L^2)$

$D = (\frac{1}{2}\,\rho a S)\, V^2 + (\frac{bW^2}{\frac{1}{2}\rho S})\,\frac{1}{V^2}$

$D = AV^2 + \frac{B}{V^2}$ where, $A = (\frac{1}{2}\rho a S)$ and $B = \frac{bW^2}{\frac{1}{2}\rho S}$



NOTE: Minimum drag occurs when the profile drag is equal to the induced drag.

2.4 MINIMUM DRAG CONDITION (D_{MIN})

Let $D = D$

$$D = D \times \frac{L}{L}$$

For steady state and level flight, L = W

$$D = W \times \frac{D}{L},$$

$$D = W \times \frac{C_D}{C_L},$$

$$D = W \times \frac{a + bC_L^2}{C_L},$$

Therefore, for drag to be minimum, $(\frac{a + bC_L^2}{C_L})$ should be minimum,

$$\mathbf{a = bC_L^2}$$

This is the condition for minimum drag. i.e.

{Parasite drag coefficient} = {induced drag coefficient}

OR, {Parasite drag} = {Induced drag}

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Example 2.1: The drag polar equation of an advanced light twin airplane is $C_D = 0.0358 + 0.0405\,C_L^2$. Its weight is 2180kg and wing area is 15m^2. Calculate:

1. Maximum lift to drag ratio

2. Minimum drag speed

3. Minimum power required with corresponding flight speed. Assume standard sea level conditions. Assume standard sea level conditions.

Solution: Given: W =2180kg = 21385.8N, S = 15m^2, $C_D = 0.0358 + 0.0405C_L^2$

1. To find Maximum lift to drag ratio

$(L/D)_{max}$ occurs under minimum drag condition, i.e. $a = b\,C_L^2$

$$\therefore C_{L_{md}} = \sqrt{\frac{a}{b}} = 0.94$$

$$C_{D_{md}} = 2a = 0.0716$$

$$\left(\frac{L}{D}\right)_{max} = \left(\frac{C_L}{C_D}\right)_{max} = \left(\frac{C_L}{C_D}\right)_{md} = \mathbf{13.13}$$

2. To find V_{md}

$$V_{md} = \sqrt{\frac{2\,W}{\rho\, s\, C_{L_{md}}}} = \sqrt{\frac{2 \times 21385.8}{1.2256 \times 15 \times 0.94}} = \mathbf{49.75\ m/s}$$

3. To find P_{Rmin}

The condition for power required to be minimum is $a = \frac{1}{3}\,b\,C_L^2$

$$\therefore C_{L_{mp}} = \sqrt{\frac{3a}{b}} = 1.6285$$

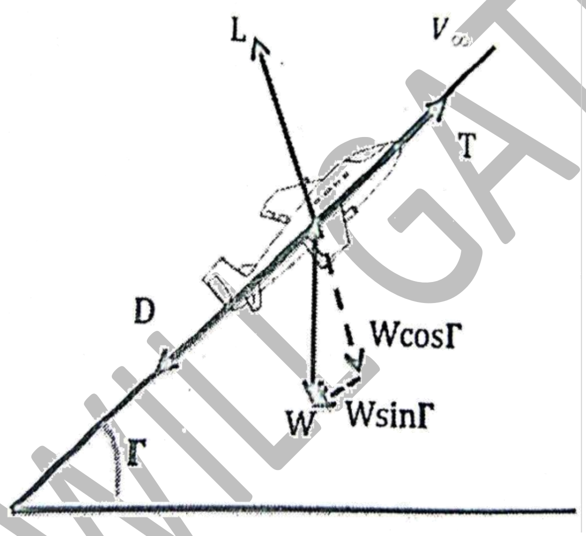
$$C_{D_{md}} = 4a = 0.1432$$

$$V_{mp} = \sqrt{\frac{2\,W}{\rho\, s\, C_{L_{mp}}}} = 37.798 \text{ m/s}$$

$$D_{mp} = W\left(\frac{C_D}{C_L}\right)_{mp} = 1880.53\text{N}$$

$$\therefore \text{ Minimum Power required}, P_{R_{min}} = D_{mp} x\, V_{mp} = \mathbf{71080.4\ watt}$$

2.14 CLIMBING FLIGHT



Consider an aircraft in steady climb along a straight flight path inclined at an angle ϒ to the horizontal, **ϒ is called as angle of climb (AOC).**

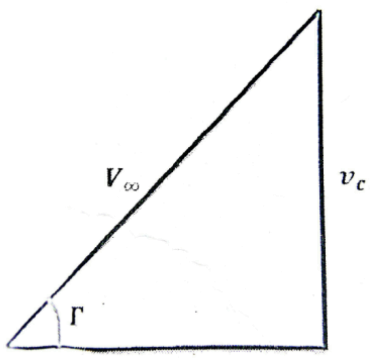
Resolving the forces,

$$L = W\cos ϒ$$

$$T = D + W \sin ϒ$$

$$\sin ϒ = \frac{T-D}{W}$$

$$ϒ = \sin^{-1}\left(\frac{T-D}{W}\right)$$



$$\boldsymbol{v_c = V_\infty Sin ϒ = V_\infty\left(\frac{T-D}{W}\right)}$$

Where, v_c is the climbing speed (or) Rate of Climb (ROC)

- ***MAXIMUM ANGLE OF CLIMB (AOC)***

$$\Upsilon = sin^{-1}(\frac{T-D}{W})$$

$$\boldsymbol{\Upsilon_{max} = sin^{-1}(\frac{T_A-D_{min}}{W})}$$

Thus for Maximum Angle of Climb, Airplane should be operated under minimum drag condition i.e. a = bC_L^2

➢ MAXIMUM RATE OF CLIMB

CASE 1: JET ENGINED AIRPLANES

$v_c = V_\infty Sin\Upsilon = V_\infty(\frac{T-D}{W})$

$v_s = \sqrt{\frac{2W}{\rho S C_L}}(\frac{T-D}{W})$

On solving, $\boldsymbol{bC_L^2 + \frac{T}{W}C_L - 3a = 0}$

This is the condition for maximum Rate of climb for Jet Engined Airplane

CASE II: PISTON ENGINED AIRPLANES-

$v_c = V_\infty Sin\Upsilon = V_\infty(\frac{T-D}{W})$

$v_c = V_\infty(\frac{T_A-D}{W})$

$v_c = (\frac{T_A V_\infty - DV_\infty}{W})$

$\boldsymbol{v_c = \left(\frac{P_A - P_R}{W}\right) = \frac{Excess\ Power\ Available}{W} = Specific\ Excess\ Power}$

$\boldsymbol{(v_c)_{max} = ROC_{max} = \frac{P_A - (P_R)_{min}}{W}}$ **= Maximum Specific Power**

Thus the Maximum Rate of climb for Piston Engined Airplane occurs under Minimum Power Condition i.e.

$$\mathbf{a = \frac{1}{3}bC_L^2}$$

Example 2.4: Estimate the rate of climb and corresponding flight speed and angle of climb if the thrust is 27000N at all speeds. Weight of an airplane is 160000N and wing area is 42m^2. The drag polar equation is $C_D = 0.014 + 0.05\ C_L^2$.

Solution: Given: W= 160000N, S = 42m^2, $C_D = 0.014 + 0.05\ C_L^2$, T = 27000N

We know, the condition for maximum rate of climb for jet engine airplane is

$bC_L^2 + \frac{T}{W} C_L - 3a = 0$

$\therefore C_L = 0.2328 \; or - 3.6078$ (neglecting negative value)

$\therefore C_D = 0.01671$(From Drag Polar equation)

$V_\infty = \sqrt{\frac{2W}{\rho \, s \, C_L}}$ = 163.412 m/s

D = W $(\frac{C_D}{C_L})$ = 11.4845 x 10^3 N

ϒ = $\sin^{-1}(\frac{T-D}{W})$ = 5.56^0

Thus, the maximum rate of climb is,

v_{Cmax} = ROC_{max} = $V_\infty \sin ϒ$ = $V_\infty(\frac{T-D}{W})$

v_{Cmax} = ROC_{max} = 15.846 m/s

GATE QUESTIONS

Q1. For a level flight at cruise altitude, C_D=0.018 with drag coefficient at zero lift, $C_{D,0}$=0.015. For a 30° climb at the same altitude and speed, C_D = ______×10^{-3}. [GATE 2015]

Ans: $\boldsymbol{C_D = 17.25 \times 10^{-3}}$

Given: C_D=0.018, $C_{D,0}$=0.015

Let $C_{L1} corresponds\ to\ lift\ coefficient\ in\ level\ flight\ and\ C_{L2}\ corresponds$

$to\ lift\ coeffcient\ in\ climb$

We know, $C_D = C_{Do} + kC_L^2$

$$0.018 = 0.015 + kC_{L1}^2$$

$$\therefore kC_{L1}^2 = 0.003$$

$$C_{L_1} = \frac{2\,W}{\rho\, s\, v^2} \qquad C_{L_2} = \frac{2\,W\,cos30}{\rho\, s\, v^2}$$

$$\therefore k = \frac{0.003}{C_L^2} = \frac{0.003}{4\,W^2}\rho^2 s^2 v^4$$

$$C_D = 0.015 + \frac{0.003}{4\,W^2}\rho^2 s^2 v^4\ \frac{4\,W^2\,3}{4\,\rho^2 s^2 v^4}$$

$$= 0.015 + 0.003\,\frac{3}{4}$$

$$\therefore \boldsymbol{C_D = 17.25 \times 10^{-3}}$$

Q2. A glider having a mass of 500 kg is taken to an altitude of 1000 m with a jeep moving on ground at 54 kmph. Upon reaching the required altitude in 50 s, the glider is released and starts its descent. Under the assumption of equilibrium glide, the range and endurance of the glider for a constant lift-to-drag ratio of 15 are **[GATE 2011]**

a) 15.0 km and 1002.2 s
b) 15.0 km and 301.3 s
c) 1.0 km and 1002.2 s
d) 1.0 km and 50 s

Ans: a) 15.0 km and 1002.2 s

Given: m = 500kg, h = 1000m, b = 54km/hr, T =50sec

Range = H ($\frac{L}{D}$) = 1000 (15) = 15000 m

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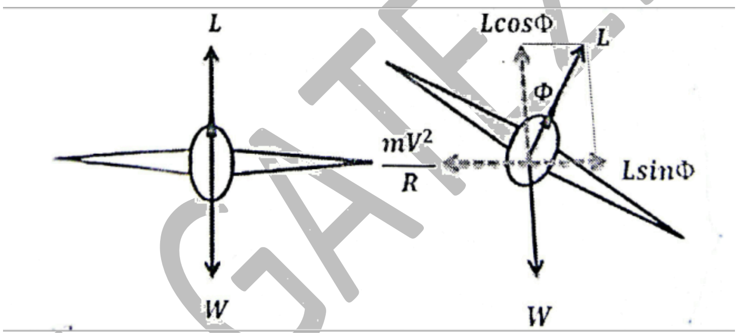
3.6 TURNING FLIGHT

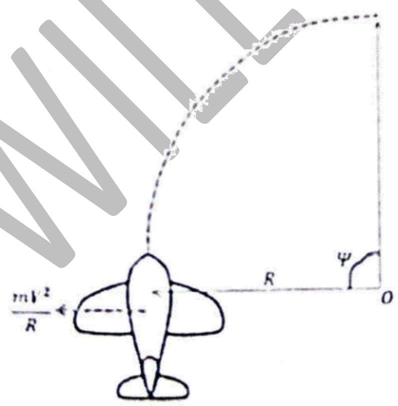
1. Turning flight, sometimes also called as **co-ordinated turn or level turn or correctly banked turn.**
2. If there is no change in altitude during turn, it is known as level turn or co-ordinated turn.

Case 1: If $\frac{mV^2}{R} = Lsin\emptyset$; it is called as **co-ordinated turn**.

Case 2: If $\frac{mV^2}{R} < Lsin\emptyset$; it is called as **side-slip**.

Case 3: If $\frac{mV^2}{R} > Lsin\emptyset$; it is called as **skidding**.





3. **The two performance characteristics of greatest importance in turning flight are**
   (i) The radius of turn
   (ii) The turn rate ω = $\frac{d\psi}{dt}$, where ψ is the turning angle.
4. Consider an airplane turning steadily at correct angle of bank around point 'O' and radius 'R' at given true air speed.

Resolving forces in the horizontal direction,

$$\frac{mV^2}{R} = Lsin\emptyset$$

Resolving forces in the vertical direction,

$$W = L\cos\emptyset$$

Dividing above equations,

$$tan\emptyset = \frac{mV^2}{R \times W} = \frac{V^2}{gR}$$

Therefore,

$$\boldsymbol{R = \frac{V^2}{gtan\emptyset}}$$

5. **Load factor (n):** It is ratio of lift of an airplane to its weight.

$$\boldsymbol{n = \frac{L}{W}}$$

But in case of turning flight, W = Lcos∅

Therefore,

$$n = \frac{L}{W} = \frac{L}{Lcos\emptyset} = \frac{1}{cos\emptyset} = sec\emptyset$$

Hence, $\boldsymbol{tan\emptyset = \sqrt{n^2 - 1}}$

We get,

$$\boldsymbol{R = \frac{V^2}{gtan\emptyset} = \frac{V^2}{g\sqrt{n^2 - 1}}}$$

Note:

a. For steady state and level flight, L=W, therefore, n = 1.
b. For climbing flight/ gliding flight
   L = Wcosγ

$$\frac{L}{W} = n = \frac{Wcos\gamma}{W}$$
$$\boldsymbol{n = cos\gamma} < 1$$

c. In steady state and level flight, before taking a turn, let T_0 be the thrust, D_0 be the drag, L_0 be the lift, V_0 be the velocity and P_0 be the power required.

For steady state and level flight, $T_0 = D_0$ and $L_0 = W$

Load factor, n = $\frac{L}{W} = \frac{L}{L0} = \frac{D}{D0}$

Assuming incidence to be constant, we can write,

$$\frac{C_L}{C_D} = \frac{L}{L0} = \frac{D}{D0}$$

$$\boldsymbol{Also, L = n}\ Lo$$

$\frac{1}{2}\rho V^2 SC_L = n\,\frac{1}{2}\rho Vo^2\ SC_L$

$V^2 = n\ Vo^2$

$\mathbf{V = \sqrt{n}\ V_O}$

$(\frac{V}{Vo})^2 = n$

Also, $P = D \times V$ and $P_0 = D_0 \times V_0$

$$\frac{P}{Po} = \frac{D \times V}{Do \times Vo} = n \times \sqrt{n} = n^{\frac{3}{2}}$$

$$P = n^{\frac{3}{2}} \times P_0$$

$$\boldsymbol{\frac{L}{L_0} = \frac{D}{D_0} = \frac{T}{T_0} = (\frac{V}{V_0})^2 = (\frac{P}{P_0})^{2/3} = n = \sec\phi}$$

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4.8 COMPLETE STABILITY EQUATION AND ITS DERIVATIVES

$M_{cg} = M_F + M_w + M_t$

$M_{cg} = M_F + M_{ac} + L(h - h_0)C - l \times \boldsymbol{L_t}$

$\boldsymbol{C_{Mcg} = C_{MF} + C_{Mac} + C_L(h - h_0) - (\alpha_w - i_w + i_t - \epsilon)\,\bar{V}\eta_t a_t}$

This is the complete stability equation for an airplane in the longitudinal mode.

NOTE: The pitching moment coefficient from fuselage is generally neglected as it is very small in comparison to te other forms, i.e. $C_{MF} = 0$

$$\boldsymbol{C_{Mcg} = C_{Mac} + C_L(h - h_0) - (\alpha_w - i_w + i_t - \epsilon)\,\bar{V}\eta_t a_t}$$

Differentiating wrt α_w

$$\boldsymbol{\frac{dC_{Mcg}}{d\alpha_w} = \frac{dC_L}{d\alpha_w}(h - h_0) - \bar{V}\eta_t a_t(1 - \frac{d\epsilon}{d\alpha_w})}$$

Important points:

1. Airfoil characteristics : The effect of airfoil characteristics is negligible if the airfoil selected for the wing and tail has constant values of the pitching moment about the aerodynamic centre, in such case only the lift curve slope is significant but that too is very small.
2. C.G. position: If the CG is ahead of AC , it contributes positively, if it coincides with C induces neutral stability and if it is behind the AC , it contributes negatively for stability.
3. Tail efficiency factor
4. Tail Volume coefficient
5. Downwash: the rate of change of downwash plays the most important role in the longitudinal stability of an airplane.

As long as the term $\frac{d\epsilon}{d\alpha w} < 1$, the airplane can be stable and once it becomes more than one, there is absolutely no chance to stabilize the airplane.

NOTE: To avoid the above phenomenon, i.e. the effect of wing downwash over the tail plane, modern airplanes are designed with high tail plane configuration. Also, this effect is not very serious for Canard configuration.

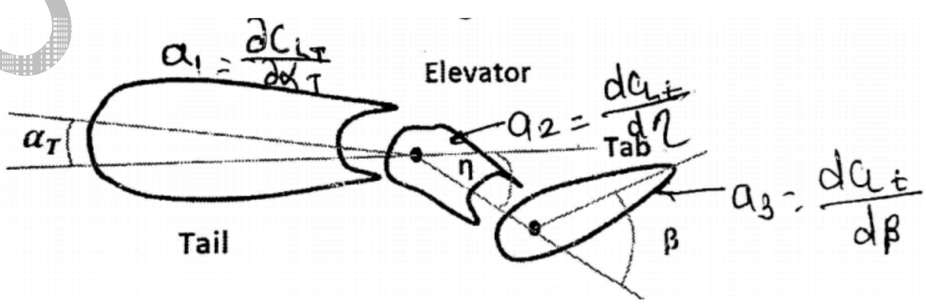
4.9 COMPLETE STABILITY EQUATION

$$C_{Mcg} = C_{MF} + C_{Mac} + C_L(h - h_0) - \bar{V}\eta_t C_{Lt}$$

Differentiating wrtC_L,

$$\frac{dC_{Mcg}}{dC_L} = \frac{dC_{MF}}{dC_L} + \frac{dC_{Mac}}{dC_L} + (h - h_0) - \bar{V}\eta_t \frac{dC_{Lt}}{dC_L}$$

$\frac{dC_{Mcg}}{dC_L} = \frac{dC_{MF}}{dC_L} + (h - h_0) - \bar{V}\eta_t \frac{dC_{Lt}}{dC_L}$---- (1)



NOTE: The downward deflection of elevator and tab is taken positive.

$$\boldsymbol{C_{Lt} = a_1\alpha_t + a_2\eta + a_3\beta}$$

$$\boldsymbol{a_1 is\ lift\ curve\ slope\ of\ the\ tail, a_1 = \frac{dC_{Lt}}{d\alpha_t}}$$

$$\boldsymbol{\alpha_t\ is\ absolute\ angle\ of\ attack\ of\ the\ tail}$$

$$\boldsymbol{a_2 is\ lift\ curve\ slope\ of\ the\ elevator},$$

$$\boldsymbol{a_2 = \frac{dC_{Lt}}{d\eta}\ ca}lled\ as\ Elevator\ effectiveness$$

η or $\boldsymbol{\delta\ is\ elevator\ deflection\ angle}$

$$\boldsymbol{a_3 = \frac{dC_{Lt}}{d\beta}\ where\ \beta\ is\ tab\ deflection\ angle}$$

Also,

$$\alpha_t = \alpha_w - i_w + i_t - \epsilon$$

$$C_{Lt} = a_1(\alpha_w - i_w + i_t - \epsilon) + a_2\eta + a_3\beta$$

Differentiating wrt $\boldsymbol{C_L}$,

$$\frac{dC_{Lt}}{dC_L} = a_1\left[\frac{d\alpha_w}{dC_L} - \frac{d\epsilon}{dC_L}\right] + a_2\frac{d\eta}{dC_L} + a_3\frac{d\beta}{dC_L}$$

$$\frac{dC_{Lt}}{dC_L} = a_1\left[\frac{d\alpha_w}{dC_L} - \frac{d\epsilon}{d\alpha_w} \times \frac{d\alpha_w}{dC_L}\right] + a_2\frac{d\eta}{dC_L} + a_3\frac{d\beta}{dC_L}$$

$$\frac{dC_{Lt}}{dC_L} = a_1\frac{d\alpha_w}{dC_L}\left[1 - \frac{d\epsilon}{d\alpha_w}\right] + a_2\frac{d\eta}{dC_L} + a_3\frac{d\beta}{dC_L}$$

$$\frac{dC_{Lt}}{dC_L} = \frac{a_1}{a}\left[1 - \frac{d\epsilon}{d\alpha_w}\right] + a_2\frac{d\eta}{dC_L} + a_3\frac{d\beta}{dC_L}$$

Substituting in equation (1)

$$\boldsymbol{\frac{dC_{Mcg}}{dC_L} = \frac{dC_{MF}}{dC_L} + (h - h_0) - \bar{V}\eta_t\{\frac{a_1}{a}\left[1 - \frac{d\epsilon}{d\alpha_w}\right] + a_2\frac{d\eta}{dC_L} + a_3\frac{d\beta}{dC_L}\}}$$

This is the complete equation for longitudinal static stability of an airplane.

4.10 STICK FIXED STABILITY

It is the condition in which the control surfaces are fixed at certain deflected position and hence,

η = constant, therefore $\frac{d\eta}{dC_L} = 0$

β = constant, therefore $\frac{d\beta}{dC_L} = 0$

therefore,

$$\left(\frac{dC_{Mcg}}{dC_L}\right)_{fixed} = \frac{dC_{MF}}{dC_L} + (h - h_0) - \bar{V}\eta_t\{\frac{a_1}{a}\left[1 - \frac{d\epsilon}{d\alpha_w}\right]\}$$

4.11 NEUTRAL POINT

1. From the equation of stability, it is observed that the amount of stability changes as the position of CG changes.
2. If the CG is moved sufficiently backwards, at some point $\frac{dC_{Mcg}}{dC_L}$ will become zero and the airplane will exhibit neutral stability, this position of CG is of a special interest and called as Neutral Point.
3. Neutral point is that special position of CG for which the slope of pitching moment coefficient versus angle of attack (or lift coefficient) is equal to zero, resulting in neutral stability of an airplane.
4. Therefore at neutral point, h = h_n and $\frac{dC_{Mcg}}{dC_L}$ =0.

4.12 STICK FIXED NEUTRAL POINT AND STICK FIXED MARGIN

We know, the stability equation for stick fixed condition is given as,

$$\frac{dC_{Mcg}}{dC_L} = \frac{dC_{MF}}{dC_L} + (h - h_0) - \bar{V}\eta_t\{\frac{a_1}{a}\left[1 - \frac{d\epsilon}{d\alpha_w}\right]\text{\_\_\_\_\_\_} \quad (1)$$

When c.g. is at neutral point, $h = h_n$ *and* $\frac{dC_{Mcg}}{dC_L} = 0$

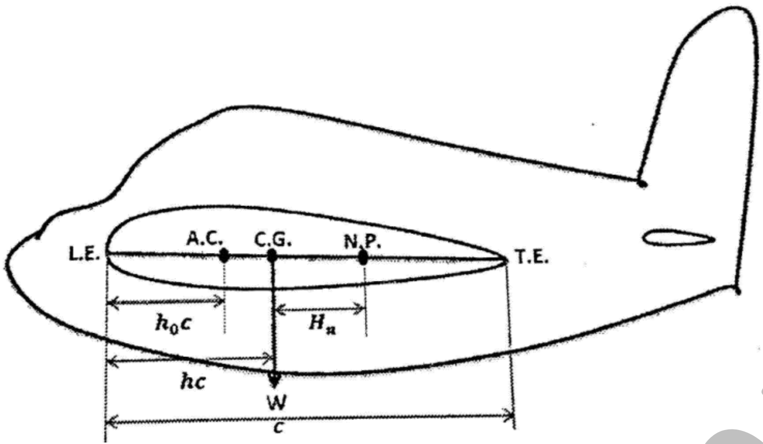
$$0 = \frac{dC_{MF}}{dC_L} + (h - h_0) - \bar{V}\eta_t\{\frac{a_1}{a}\left[1 - \frac{d\epsilon}{d\alpha_w}\right]\}\text{---} \ (2)$$

Subtracting (2) from (1), we get

$$\frac{dC_{Mcg}}{dC_L} = h - h_n$$

$$\frac{dC_{Mcg}}{dC_L} = -(h_n - h) = -H_n$$

Where, H_n is called as static margin or stick fixed static margin and it is a direct measure of static longitudinal stability of an airplane.



Static margin H_n is a distance by which the CG is ahead of the neutral point expressed as fraction of chord or in terms of chord.

4.15 LONGITUDINAL CONTROL

Control of the pitch attitude of the airplane can be achieved by deflecting the elevator.

$$\boldsymbol{C_{Mcg} = C_{MF} + C_{Mac} + C_L\left(\frac{X_a}{c}\right) - \bar{V}\eta_t\alpha_t\left(\frac{dC_L}{d\alpha}\right)_t} \text{ --------- } \textbf{(a)}$$

Elevator power: Deflecting the elevator effectively changes an angle of attack of whole horizontal tail plane, thereby changing its lift and producing a controlling moment about airplane C.G.

The magnitude of the moment coefficient obtained per degree deflection of the elevator is called as elevator power.

$$C_{m\delta e} = \frac{dC_M}{d\delta e}$$

Differentiating equation (a) with respect to δ_e

$$\frac{dC_{Mcg}}{d\delta e} = -\bar{V}\eta_t\left(\frac{dC_L}{d\alpha}\right)_t \frac{d\alpha_t}{d\delta e}$$

Where, $\frac{d\alpha_t}{d\delta e} = \tau = rate\ of\ change\ of\ \alpha_t\ with\ respect\ to\ \delta_e$

$\left(\frac{dC_L}{d\alpha}\right)_t = a_t = lift\ curve\ slope\ of\ tail$

$$\boldsymbol{C_{m\delta e} = -\bar{V}\eta_t a_t \tau}$$

This is known as elevator control power.

$$\boldsymbol{C_{m\delta e} = -\eta_t \frac{S_t}{S}\frac{l}{c}\frac{dC_{Lt}}{d\delta_e}}$$

Where $\frac{dC_{Lt}}{d\delta_e} = \boldsymbol{elevator\ effectiveness}$

NOTE:

- Elevator effectiveness or elevator power, controls the forward most location of C.G.
- Neutral point restricts the aft most position of C.G.

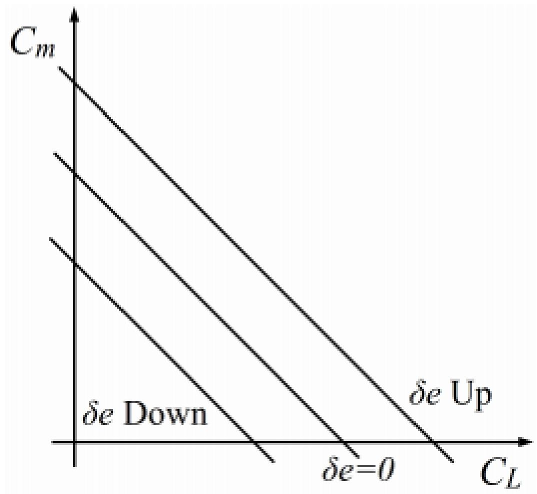
The only term affected by elevator deflection is the tail angle of attack (α_t)

$$\alpha_t = \alpha_w - i_w + i_t - \epsilon + \tau\delta_e$$

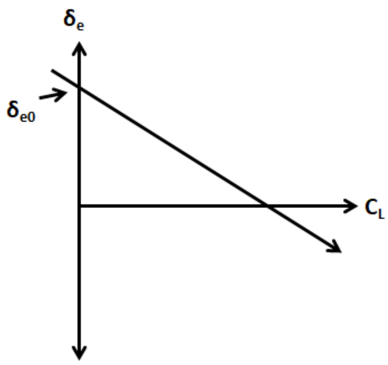
$C_{Mcg} = C_{MF} + C_{Mac} + C_L\left(\frac{X_a}{c}\right) - \bar{V}\eta_t(\alpha_w - i_w + i_t - \epsilon + \tau\delta_e)\, a_t$---------- (b)

Since τ is independent of the lift coefficient C_L, if the equation (b) is differentiated with respect to C_L, the term $\tau\delta_e$ vanishes, which means, **the change in elevator deflection will not change the slope of the pitching moment curve, i.e.** $\frac{dC_M}{dC_L}$

Thus the deflection of elevator will only change the equilibrium lift coefficient (C_L *corresponding to* α_{trim}) but will not change the longitudinal static stability.



$C_M = C_{Mo} + \frac{dC_M}{d\alpha}\alpha + \frac{dC_M}{d\delta_e}\delta_e$



Elevator angle to trim

$$\delta_{e\,trim} = \delta_{e0} + \frac{d\delta_e}{dC_{L\,trim}}C_{L\,trim}$$

Where, $\boldsymbol{\delta_{e0}\ is\ the\ elevator\ angle\ at\ zero\ lift}$

This equation can be used to estimate the value of elevator deflection required to trim a given aircraft at a particular $C_{L\,trim}$

$$\frac{dC_M}{dC_L} = -C_{M\delta e}\frac{d\delta_e}{dC_{L\,trim}}$$

$$\frac{dC_M}{dC_L} = -(-\bar{V}\eta_t a_t \tau)\frac{d\delta_e}{dC_{L\,trim}}$$

$$\frac{dC_M}{dC_L} = (\bar{V}\eta_t a_t \tau)\frac{d\delta_e}{dC_{L\,trim}}$$

$$\frac{dC_M}{dC_L} = (k)\frac{d\delta_e}{dC_{L\,trim}}$$

Therefore, the criteria for static longitudinal stability is, $\frac{dC_M}{dC_L} < 0$, the same criteria can be represented in terms of $\frac{d\delta_e}{dC_{L\,trim}} < 0$

4.16 STICK FIXED MANEUVERING POINT

Stick fixed manoeuvring point is the C.G. location at which $\frac{d\delta_e}{dn} = 0$

$$\overline{N_M} = \overline{N_0} - \frac{1.1 g l_t \rho C_{m\delta e}}{2\tau\left(\frac{W}{S}\right)}\left(1 + \frac{K'}{n^2}\right)$$

Where $\overline{N_M}$ *and* $\overline{N_0}$ are the non dimensional locations of the stick –fixed maneuver point and stick – fixed neutral point respectively.

As $C_{m\delta e} < 0$, hence **stick fixed Manoeuvring point is aft of stick fixed neutral point.**

$$\frac{d\delta_e}{dn} = -\frac{1}{V^2}\frac{2\left(\frac{W}{S}\right)}{\rho C_{M\delta e}}(\overline{x_{cg}} - \overline{N_M})$$

Where, $\overline{N_M} - \overline{x_{cg}}$ *is called maneuvering margin*

- The derivative dδe/dn varies with maneuver margin.
- For more forward C.G. location, more elevator will be required to obtain the limit load factor. Therefore, as the C.G. moves forward, more elevator deflection is necessary to obtain a given load factor.
- The lower positive speed (higher the C_L) more elevator will be necessary to set the limit load factor. Thus, at low speeds more elevator deflection is necessary to obtain a desired load factor than is required to obtain at a higher speed.

4.23 CONTRIBUTION OF WING SWEEP

$$C_{L\beta} = \frac{dC_L}{d\beta} = -C_L\left(\frac{y}{b}\right)\sin 2\lambda$$

Where λ is the sweep angle

- It is seen that the dihedral effect due to sweep back is directly proportional to the lift coefficient C_L, side slip angle β and sweep angle λ.
- The sweep back wing provides a dihedral effect.
- Since λ is positive for swept back wings, sin2λ is positive and $C_{L\beta}$ is negative which is a positive dihedral effect; hence sweep back wing provides stabilizing effect in the lateral mode.
- For swept forward wing, λ will be negative and therefore $C_{L\beta}$ will be positive, which is a negative dihedral effect, hence sweep forward wing provides de - stabilizing effect in the lateral mode.

NOTE:

- A high wing on fuselage with negative dihedral angle stabilizes the DUTCH ROLL mode.
- A low wing with the dihedral gives highest restoring roll moment against side slip. It stabilizes SPIRAL MODE and hence has large roll stability.

GATE QUESTIONS

Q1. A jet aircraft is initially flying steady and level flight at its maximum endurance condition. For the aircraft to fly steady and level, but faster at the same altitude, the pilot should **[GATE 2018]**

(a) increase thrust alone

(b) increase thrust and increase angle of attack

(c) increase thrust and reduce angle of attack

(d) reduce angle of attack alone

ANS: (c) increase thrust and reduce angle of attack

To move faster the pilot has to increase the thrust of an aircraft. But as thrust increases, velocity increases, which in turn increases the lift. As usually the aerodynamic centre is ahead of the c.g. of an aircraft, it will produce a nose up pitching moment. To maintain steady and level flight, the pilot has to decrease angle of attack to compensate for the additional lift.

Q2. A statically – stable aircraft has a $C_{L\alpha} = 5$ (were the angle of attack, α, is measured in radians). The coefficient of moment of the aircraft about the centre of gravity is given as $C_{M,c.g.} = 0.05 - 4\alpha$. The mean aerodynamic chord of the aircraft wing is 1m. The location (positive towards the nose) of the neutral point of the aircraft from the centre of gravity is ____(in m, accurate to two decimal places). **[GATE 2018]**

Ans : - 0.8

Given: $\frac{dC_L}{d\alpha}$ = 5 /radians, $C_{M,c.g.} = 0.05 - 4\alpha$, c = 1m

We know, $\boldsymbol{\frac{dC_{Mcg}}{dC_L} = h - h_n}$

$\boldsymbol{where\ h\ is\ the\ chord\ wise\ location\ of\ C.G.\ and\ h_n\ is\ the\ chord\ wise\ location\ of\ neutral\ point}$

$$\frac{dC_{Mcg}}{d\alpha} = -4$$

Q11. The trim curves of an aircraft are of the form $C_{m,0} = (0.05 - 0.2\delta_e) - 0.1C_L$, where the elevator deflection angle δ_e is in radians. The static margin of the aircraft is: **[GATE 2010]**

(a) 0.5 (b) 0.2 (c) 0.1 (d) 0.05

Ans: (c) 0.1

$$C_{MCG} = (-0.05 - 0.2\delta_e) - 0.1\ C_L$$

$$\frac{dC_{MCG}}{dC_L} = 0.2\ \frac{d\delta_e}{dC_L} - 0.1$$

For trim, $\frac{dC_M}{dC_L} = 0 = \frac{dC_M}{d\delta_e}\ \frac{d\delta_e}{dC_L}$

$$\therefore\ \frac{d\delta_e}{dC_L} = 0$$

$$\therefore \frac{dC_{MCG}}{dC_L} = -H_n = -0.1$$

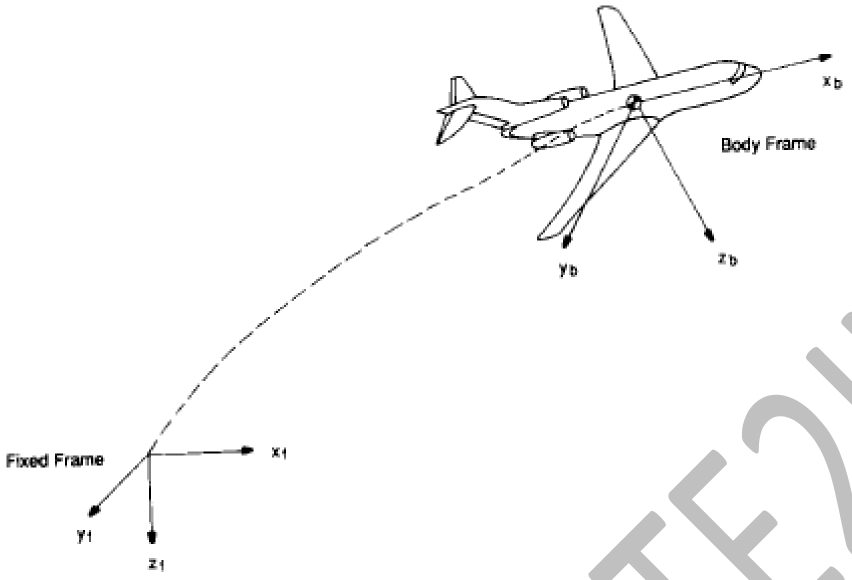
$$\therefore \boldsymbol{H_n} = \mathbf{0.1}$$

Chapter 6

EQUATIONS OF MOTION

6.1 Body - fixed Coordinate system

Thus coordinate system is having origin at the center of gravity of the aircraft in which x-axis has been taken along the length of the aircraft i.e. along the length of fuselage in the airplane's plane of symmetry; y-axis has been taken as perpendicular to the aircraft's plane of symmetry. It has been denoted as (x_b, y_b, z_b) as shown below.



6.4 EULER ANGLE FORMULATION

The position of the aircraft is specified by the location of the origin of the body - fixed coordinate frame relative to the Earth – fixed coordinate frame.

- The orientation of an aircraft relative to the Earth can be described in terms of what are called **EULER ANGLES**.
- The orientation of the body fixed coordinate frame (x_b, y_b, z_b) relative to the Earth fixed coordinate frame (x_f, y_f, z_f) is described in terms of three consecutive rotations through three Euler angles, in the specific order as follows –
    - ➢ Rotate the (x_f, y_f, z_f) coordinate system about Z_f axis through an angle Ψ to the coordinate system (x_1, y_1, z_1)
    - ➢ Rotate the (x_1, y_1, z_1) coordinate system about y_1 axis through an angle Θ to the coordinate system (x_2, y_2, z_2)
    - ➢ Rotate the (x_2, y_2, z_2) coordinate system about x_2 axis through an angle ϕ to the coordinate system (x_b, y_b, z_b)
- The three Euler angles ϕ, Θ and Ψ are respectively, called the bank angle, the elevation angle and the azimuth angle or heading.
- There is a subtle but very important difference between the Euler angles and roll, pitch and yaw.
- Roll, Pitch and Yaw are orthogonal whereas, the Euler's angles are not.

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