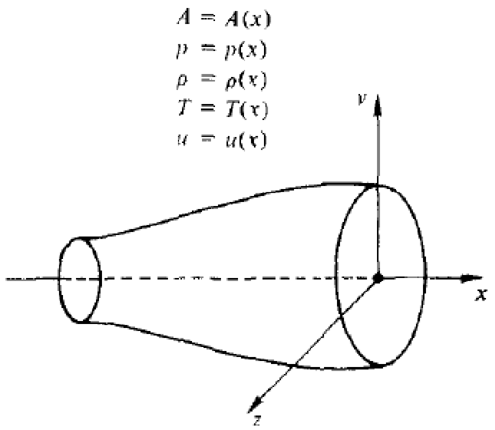
# GAS DYNAMICS

## *QUASI ONE DIMENSIONAL FLOW*

A flow, where the area varies as A = A(x) but it is assumed that p, ρ, T and u are still functions of x only, is defined as quasi-one-dimensional flow.




## *CONSERVATION EQUATIONS*

1. **Continuity Equation:**

$$\rho_1 A_1 u_1 = \rho_2 A_2 u_2$$

2. **Momentum equation**

$$p_1 A_1 + \rho_1 A_1 u_1^2 = p_2 A_2 + \rho_2 A_2 u_2^2$$

3. **Energy equation:**

$$h_1 + \frac{u_1^2}{2} = h_2 + \frac{u_2^2}{2}$$

## *SPEED OF SOUND*

$$a = \sqrt{\frac{dp}{d\rho}}$$

For incompressible flow, speed of sound is infinite.

$$a = \sqrt{\frac{\gamma p}{\rho}} = \sqrt{\gamma RT}$$

The speed of sound in air at standard sea level conditions is

a = 340.9 m/s

Mach number is defined as M = V/a, which leads to the following classification of different flow regimes:

M < 1 (Subsonic flow)

M = 1 (Sonic flow)

M > 1 (Supersonic flow)

## STAGNATION STATE

Stagnation state is obtained by decelerating a gas isentropically to zero velocity at zero elevation.

**Stagnation Enthalpy**

$$h_0 = h + \frac{c^2}{2}$$

**Stagnation temperature**

$$T_0 = T + \frac{c^2}{2C_p}$$

$$\frac{T_0}{T} = 1 + \frac{\gamma - 1}{2} M^2$$

**Stagnation Pressure**

For isentropic flow,

$$\frac{P_2}{P_1} = \left(\frac{T_2}{T_1}\right)^{\frac{\gamma}{\gamma-1}}$$

$$\frac{P_0}{P} = \left(1 + \frac{\gamma - 1}{2} M^2\right)^{\frac{\gamma}{\gamma-1}}$$

**Stagnation Density**

For isentropic flow,

$$\frac{\rho_2}{\rho_1} = \left(\frac{T_2}{T_1}\right)^{\frac{1}{\gamma-1}}$$

$$\frac{\rho_0}{\rho} = \left(1 + \frac{\gamma - 1}{2} M^2\right)^{\frac{1}{\gamma-1}}$$

**Stagnation velocity of sound**

$$a_0 = \sqrt{\gamma R T_0}$$

## CHOKED FLOW

A flow which has a maximum mass flow rate is known as choked flow. Flows will choke at area minima in a duct.

- M = 1
- Mass flow rate is maximum
- Properties are represented as $P^*, T^*, \rho^*, a^*, A^*$

$$\frac{T_0}{T^*} = \frac{\gamma + 1}{2}$$

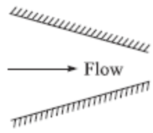
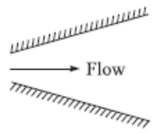
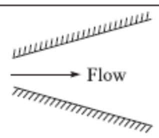
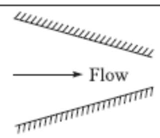
$$\frac{P_0}{P^*} = \left(\frac{\gamma+1}{2}\right)^{\frac{\gamma}{\gamma-1}}$$

$$\frac{\rho_0}{\rho^*} = \left(\frac{\gamma+1}{2}\right)^{\frac{1}{\gamma-1}}$$

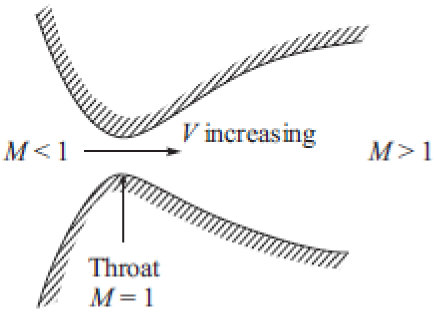
## AREA VELOCITY RELATION

$$\boldsymbol{\frac{dA}{A} = \frac{dV}{V}(M^2 - 1)}$$

- For $0 \le M \le 1$, a decrease in area results in increase of velocity, and vice versa. Therefore, the velocity increases in a convergent duct and decreases in a divergent duct. This result for compressible subsonic flows is the same as that for incompressible flow.
- For $M > 1$, an increase in area results in increase of velocity and vice versa, i.e. the velocity increases in a divergent duct and decreases in a convergent duct. This is directly opposite to the behaviour of subsonic flow in divergent and convergent ducts.
- For $M = 1$, by Eq. (4.27), $dA/A = 0$, which implies that the location where the Mach number is unity, the area of the passage is either minimum or maximum. The minimum area is the only physically realistic solution.

| Velocity, $V$ | Subsonic, $M < 1$ | Supersonic, $M > 1$ |
|---|---|---|
| Increases |   |   |
| Decreases |   |   |

- From the above discussions, it is clear that for a gas to expand isentropically from subsonic to supersonic speeds; it must flow through a convergent divergent duct, as shown in Figure. The minimum area that divides the convergent and divergent sections of the duct is called the *throat*. From point 4 above, we know that the flow at the throat must be sonic with $M$ = 1.
- Conversely, for a gas to get compressed isentropically from supersonic to subsonic speeds, it must again flow through a convergent-divergent duct, with a throat where sonic flow occurs.

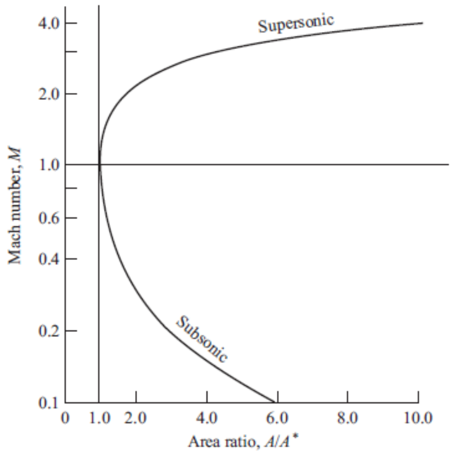



## AREA MACH RELATION

$$\frac{A}{\boldsymbol{A^*}} = \frac{\boldsymbol{1}}{\boldsymbol{M}}\left[\frac{2}{\gamma + 1}\left(1 + \frac{\gamma - 1}{2}M^2\right)\right]^{\frac{\gamma+1}{2(\gamma-1)}}$$

- The Mach number at any location in the duct is a function of the ratio of the local area of the duct to the sonic throat area.
- The local duct area, A, must be larger than or at least equal to $A^*$, the case in which A < $A^*$ is physically impossible in an isentropic flow.

- For any given $A/A^* > 1$, two values of M are obtained: a subsonic value and a supersonic value.




## CHOCKED MASS FLOW RATE OF THE NOZZLE

The mass flow rate of the nozzle remains unaltered after flow gets chocked. This chocked mass flow rate can be calculated as

$$\dot{m} = \rho^* u^* A^*$$

But we know that,

$$\frac{\rho_0}{\rho^*} = \left[\frac{(\gamma+1)}{2}\right]^{\frac{1}{(\gamma-1)}}$$

$$u^* = a^* = \sqrt{\gamma R T^*}$$

Hence

$$\dot{m} = \rho_0 \left[\frac{(\gamma+1)}{2}\right]^{\frac{-1}{(\gamma-1)}} \sqrt{\gamma R T^*} A^*$$

$$\dot{m} = \rho_0 \left[\frac{(\gamma+1)}{2}\right]^{\frac{-1}{(\gamma-1)}} \sqrt{\gamma R \frac{T^*}{T_0} T_0} A^*$$

$$\rho_0 = \frac{p_0}{R T_0}$$

$$\frac{T^*}{T_0} = \left[1 + \frac{(\gamma-1)}{2}\right]^{-1}$$

Hence

$$\dot{m} = \frac{p_0}{R T_0} \left[\frac{(\gamma+1)}{2}\right]^{\frac{-1}{(\gamma-1)}} \sqrt{\gamma R \frac{T^*}{T_0} T_0} A^*$$

$$\dot{m} = p_0 \left[\frac{(\gamma+1)}{2}\right]^{\frac{-(\gamma+1)}{2(\gamma-1)}} \sqrt{\frac{\gamma}{R T_0}} A^*$$

From this expression it is clear that for a convergent divergent nozzle, for given throat area, choked mass flow rate remains constant for the fixed reservoir ($P_0$ and $T_0$) conditions. Therefore choked mass flow rate can be increased by increasing the reservoir pressure $P_0$ or decreasing reservoir temperature $T_0$.

## *CHARACTERISTIC MACH NUMBER*

It is the ratio of local flow speed and critical speed of sound,

$$\boldsymbol{M^{*2}} = \frac{\boldsymbol{M^2 (\frac{\gamma+1}{2})}}{\boldsymbol{1 + \frac{\gamma-1}{2} M^2}}$$

It can be observed:

$$M^* = 1 \; if \; M = 1$$

$$M^* < 1 \; if \; M < 1$$

$$M^* > 1 \; if \; M > 1$$

$$M^* \to \sqrt{\frac{\gamma+1}{\gamma-1}} \; if \; M \to 1$$

Hence, qualitatively, $M^*$ behaves in the same manner as $M$, except when $M$ goes to infinity. $M^*$ will be a useful parameter for further building of the subject involving shocks and expansion waves because it approaches a finite value as $M$ approaches infinity.