GOODWILL

GATE2IIT

IIT-JEE | MEDICAL | GATE AEROSPACE

AIRCRAFT STRUCTURES

GATE AEROSPACE

GuneetKaur Chadha

GOODWILL GATE2IIT

IIT-JEE | MEDICAL | GATE

GATE 2019

ORGANIZING INSTITUTE :

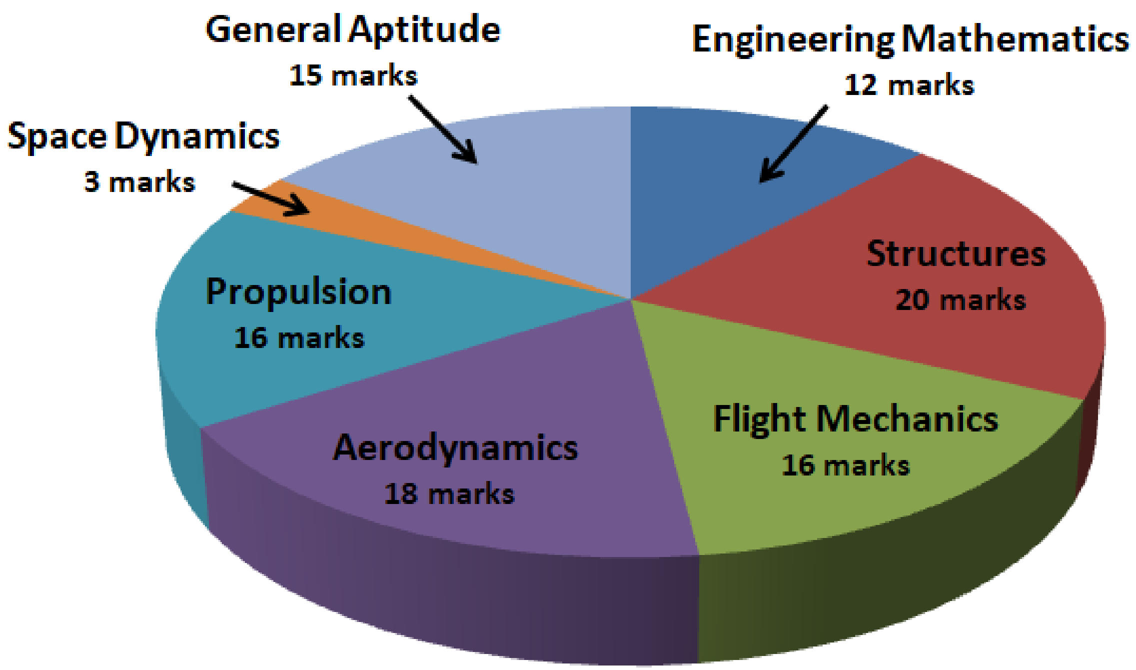
IIT MADRAS

AEROSPACE ENGINEERING

Join **GATE AEROPSACE DISCUSSSION FORUM** on Facebook and clear all your doubts

Analysis of GATE

AEROSPACE ENGINEERING



NOTE: *Above values coresponds to general distribution of marks for the following subjects. GATE subject wise marks every year varies from 1 to 2 marks from above values.*

GATE AEROSPACE Year Wise Analysis

Subject	No of Questions	Topics	Total Marks
Engineering Mathematics	1M : 3 2M : 5	• Matrices • Differential and Integral Calculus • Vector calculus • ODE / PDE • Numerical Technique • Laplace transformation • Fourier	13
Flight Mechanics	1M : 7 2M : 5	• Atmosphere • Basics – Speeds, Altitudes and primary flight instruments • Steady straight and level flight • Gliding and Climbing • Range and Endurance • Take – off and Landing • Turning flight • Pull –up and pull – down manoeuvre • V-n diagram • Longitudinal static stability • Directional and Lateral static stability • Dynamic Stability - modes • Equations of motion • Euler's angles	17
Space Dynamics	1M : 1 2M : 1	• Kepler's Law • Different orbits – circular, elliptic, parabolic and hyperbolic	3 (in 2018 GATE AE Space mechanics covers 4

		• Escape velocity • Orbit in-plane transfer (Hohmann) • Orbit out–off plane transfer	questions – 5 marks)
Aerodynamics	1M : 3 2M : 6	• Basic Fluid mechanics – Laminar and turbulent flow, Boundary layer, Dimensional similarity • Ideal potential flow • Low speed aerodynamics – aerodynamic centre, center of pressure • Thin Airfoil theory • Finite wing theory • Gas Dynamics – Isentropic flow, CD nozzles, NSW, OSW, Expansion fans, Rayleigh flow, Fanno flow	15
Structures	1M : 5 2M : 7	• Basic Elasticity – Stress tensor, Principal stresses, Mohr's circle, Generalized Hooke's law, Plane stress, plane strain, Airy's stress function, Thin walled pressure vessels • Bending- Symmetrical and unsymmetrical bending, bending stresses, shear stresses, S.F. and B.M. diagram, deflection of beams, Shear flow and Shear centre, Structure idealization • Torsion of thin walled sections • Columns	19

		• Theory of failures • Vibration – Damped and undamped, free and forced system 1 DOF system, Free 2 DOF system, Continuous vibration • Energy methods	
Propulsion	1M : 6 2M : 6	• Thermodynamics Basics • Axial flow compressor • Axial flow turbine • Centrifugal flow compressor • Jet propulsion – Turbojet Turbofan, Ramjet, Turboprop • Combustion chamber • Rocket Propulsion	18
General Aptitude	1M:5 2M:5	• Verbal Ability • Numerical Ability	15
Total	**65**		**100**

Structures Syllabus

Core Topics:

Strength of Materials: States of stress and strain. Stress and strain transformation. Mohr's Circle. Principal stresses. Three-dimensional Hooke's law. Plane stress and strain; Failure theories: Maximum stress, Tresca and von Mises; Strain energy. Castigliano's principles. Analysis of statically determinate and indeterminate trusses and beams. Elastic flexural buckling of columns.

Flight vehicle structures: Characteristics of aircraft structures and materials. Torsion, bending and flexural shear of thin-walled sections. Loads on aircraft.

Structural Dynamics: Free and forced vibrations of undamped and damped SDOF systems. Free vibrations of undamped 2-DOF systems.

Special Topics:

Vibration of beams. Theory of elasticity: Equilibrium and compatibility equations, Airy's stress function.

Structures Year Wise Analysis

Year	No of Questions	Topics (1 marks + 2 marks)	Total Marks
2018	1M : 3 2M : 7	• Basic Elasticity (0 + 3) • Bending- (0 + 3) • Torsion (1 + 0) • Vibration (2 + 1)	17
2017	1M : 6 2M : 7	• Basic Elasticity (1 + 2) • Bending- (1 + 2) • Torsion (0 + 1) • Columns (1 + 0) • Vibration (3 + 2)	21
2016	1M : 5 2M : 6	• Basic Elasticity (3 + 2) • Bending- (1 + 1) • Columns (0 + 1) • Vibration (1 + 2)	17
2015	1M : 5 2M : 6	• Basic Elasticity (3 + 1) • Bending- (0 + 2) • Torsion (0 + 1) • Vibration (2 + 2)	17
2014	1M : 5 2M : 6	• Basic Elasticity (2 + 2) • Bending- (1 + 1) • Torsion (0 + 1) • Columns (0 + 1) • Vibration (2 + 1)	17
2013	1M : 4 2M : 7	• Basic Elasticity (0 + 1) • Bending- (1 + 3) • Torsion (2 + 1) • Columns (0 + 1) • Vibration (1 + 1)	18
2012	1M:4 2M:7	• Basic Elasticity (2 + 2) • Theory of failures (0 + 2) • Vibration (2 + 3)	18
2011	1M:6 2M:9	• Basic Elasticity (2 + 2) • Bending- (2 + 2)	24

		• Torsion (0 + 2) • Columns (0 + 1) • Vibration (2 + 2)	
2010	1M:5 2M:7	• Basic Elasticity (2 + 2) • Bending- (0 + 3) • Theory of Failures (1 + 0) • Columns (1 +0) • Vibration (1 + 2)	19
2009	1M:4 2M:8	• Basic Elasticity (1 + 0) • Bending- (2 + 2) • Torsion (0 + 1) • Columns (0 + 1) • Vibration (1 + 4)	20
2008 (85 questions)	1M:4 2M:12	• Basic Elasticity (3 + 3) • Bending- (0 + 3) • Torsion (0 + 2) • Theory of Failures (0 + 1) • Columns (0 + 1) • Vibration (1 + 2)	28 (Total 150 marks)
2007 (85 questions)	1M:3 2M:12	• Basic Elasticity (1 + 0) • Bending- (0 + 2) • Torsion (0 + 2) • Theory of Failures (0 + 1) • Columns (0 + 1) • Vibration (2 + 6)	27 (Total 150 marks)

TABLE OF CONTENT

- GATE and Additional questions
- Exercise at the end of each Chapter
- Previous year GATE questions solved

Chapter 1

BASIC ELASTICITY

1.1 STRESS

Stress is defined as the intensity of internal resisting force developed/induced at a point against the deformation causes due to loads acting on the member.

- It uses original cross section area of the specimen and also known as engineering stress or conventional stress.
  Therefore, $\sigma = \frac{P}{A}$

- P is expressed in Newton (N) and A, original area, in square meters (m2), the stress σ will be expresses in N/ m2. This unit is called Pascal (Pa).

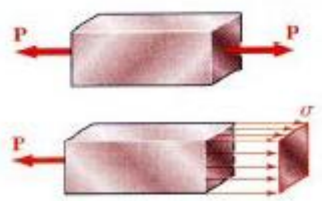
- As Pascal is a small quantity, in practice, multiples of this unit is used.
  1 kPa = 103 Pa = 103 N/ m^2 (kPa = Kilo Pascal)
  1 MPa = 106 Pa = 106N/ m^2 = 1 N/mm2 (MPa = Mega Pascal)
  1 GPa = 109Pa = 109 N/ m^2 (GPa = Giga Pascal)

- The resultant of the internal forces for an axially loaded member is normal to a section cut perpendicular to the member axis..

- The force intensity on the shown section is defined as the normal stress. $\lim_{\Delta A \to 0} \frac{\Delta F}{\Delta A}$ $and\ \sigma_{avg} = \frac{P}{A}$

- Stresses are not vectors because they do not follow vector laws of addition. They are Tensors. Stress, Strain and Moment of Inertia are second order tensors.

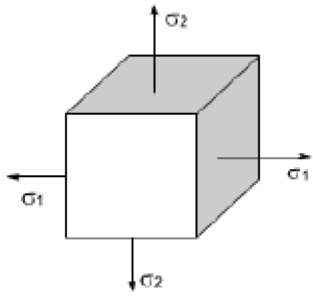
- ***Tensile stress (σ_t)***



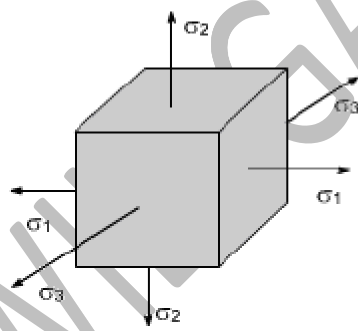
If σ > 0 the stress is tensile i.e. the fibres of the component tend to elongate due to the external force. A member subjected to an external force tensile P and tensile stress

----

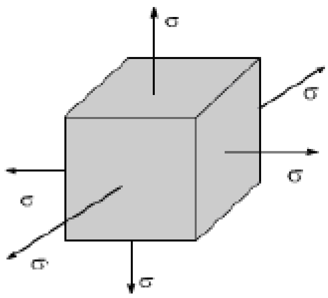
---------------------------------Pages missing in sample copy-----------------------------------
----



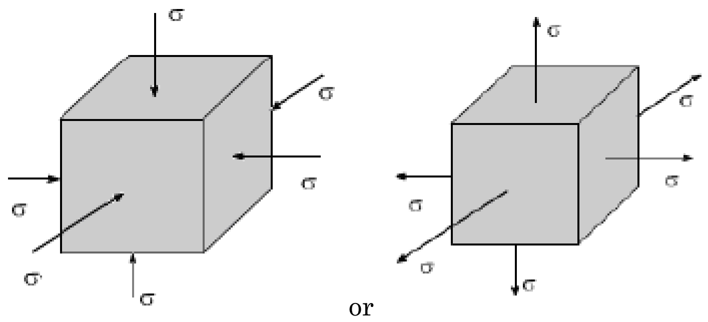
(iii) Tri-axial stress: three non-zero principal stresses, i.e. $\sigma_1 \neq 0, \sigma_2 \neq 0, \sigma_3 \neq 0$



(iv) Isotropic stress: three principal stresses are equal, i.e. $\sigma_1 = \sigma_2 = \sigma_3$



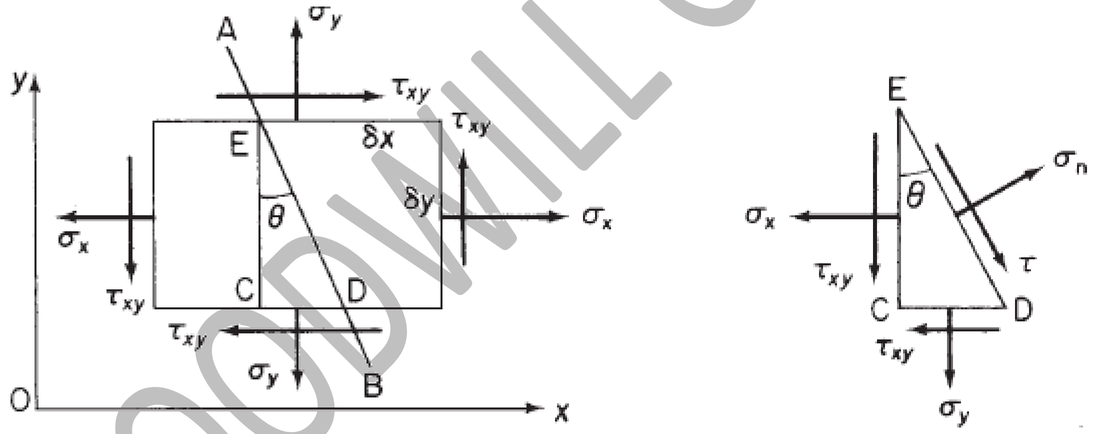
(v) Hydrostatic stress: Hydrostatic stress is used to describe a state of tensile or compressive stress equal in all directions within or external to a body. Hydrostatic stress causes a change in volume of a material. Shape of the body remains unchanged i.e. no distortion occurs in the body.



----------------------------------Pages missing in sample copy--------------------------------------
----

1.7 STRESS TRANSFORMATON

Consider a biaxial rectangular element of an elastic body as shown in the figure. Here, δx and δy are very small so the distribution of stresses along the sides are uniform.



Here, the body forces are neglected so the contribution is of second order. As rectangular element is in equilibrium under given axial loading, then rectangular element ABC will also be in equilibrium under the given loading conditions.

From force polygon diagram:

Resolving forces in σ_n and τ, we get

$$\sigma_n = \tau_{xy} \sin 2\theta + \sigma_{xx} \cos^2\theta + \sigma_{yy} \sin^2\theta$$

$$\boldsymbol{\sigma} = \left(\frac{\sigma_{xx} + \sigma_{yy}}{2}\right) + \left(\frac{\sigma_{xx} - \sigma_{yy}}{2}\right) \cos 2\theta + \tau_{xy} \sin 2\theta$$

$$\tau = \frac{\sigma_{xx}-\sigma_{yy}}{2} \sin 2\theta - \tau_{xy}\cos 2\theta$$

NOTE: Above two transformation equations are coming from considering equilibrium. They do not depend on material properties and are valid for elastic and in elastic behaviour.

1.8 PRINCIPAL STRESS AND PRINCIPAL PLANE

- **Principal stresses** are normal stresses that are orthogonal to each other
- **Principal planes** are the planes across which principal stresses act (faces of the cube) for principal stresses (shear stresses are zero)

For principal plane, τ = 0

Therefore,

$$\theta_1 = \frac{1}{2}\tan^{-1}\left(\frac{2\tau_{xy}}{\sigma_x - \sigma_y}\right)$$

This angle represents position of principal plane.

And,

$$\theta_2 = \frac{\pi}{2} + \frac{1}{2}\tan^{-1}\left(\frac{2\tau_{xy}}{\sigma_x - \sigma_y}\right)$$

Where θ_1 and θ_2 shows the direction of principal planes.

Substituting value of θ_1, we have

$$\textit{Major principal stress} = \sigma_I = \left(\frac{\sigma_{xx} + \sigma_{yy}}{2}\right) + \sqrt{\left(\frac{\sigma_{xx} - \sigma_{yy}}{2}\right)^2 + \tau_{xy}^2}$$

$$\textit{Minor principal stress} = \sigma_{II} = \left(\frac{\sigma_{xx} + \sigma_{yy}}{2}\right) - \sqrt{\left(\frac{\sigma_{xx} - \sigma_{yy}}{2}\right)^2 + \tau_{xy}^2}$$

$$\tau_{max} = \left(\frac{\sigma_I - \sigma_{II}}{2}\right) = \sqrt{\left(\frac{\sigma_{xx} - \sigma_{yy}}{2}\right)^2 + \tau_{xy}^2}$$

The plane at which τ_{max} occurs at $\theta = \theta_1 + \pi/4$

Question 1: A structural member supports loads which produce, at a particular point, a direct tensile stress of 80 N/mm^2 and a shear stress of 45 N/mm^2 on the same plane.

Calculate the values and directions of the principal stresses at the point and also the maximum shear stress, stating on which planes this will act.

Solution:

$$\sigma_x = 80\ N/mm^2\ , \sigma_y = 0\ or\ vice\ versa$$

$$\tau_{xy} = 45\ N/mm^2$$

Principal stresses are given by

$$\boldsymbol{Major\ principal\ stress = \sigma_I = \left(\frac{\sigma_{xx} + \sigma_{yy}}{2}\right) + \sqrt{\left(\frac{\sigma_{xx} - \sigma_{yy}}{2}\right)^2 + \tau_{xy}{}^2}}$$

$$\sigma_I = \left(\frac{80}{2}\right) + \sqrt{\left(\frac{80}{2}\right)^2 + 45^2} = \mathbf{100.2\ N/mm^2}$$

$$\boldsymbol{Minor\ principal\ stress = \sigma_{II} = \left(\frac{\sigma_{xx} + \sigma_{yy}}{2}\right) - \sqrt{\left(\frac{\sigma_{xx} - \sigma_{yy}}{2}\right)^2 + \tau_{xy}{}^2}}$$

$$\sigma_{II} = \left(\frac{80}{2}\right) - \sqrt{\left(\frac{80}{2}\right)^2 + 45^2} = \mathbf{-20.2\ N/mm^2}$$

The directions of the principal stresses are defined by the angle Θ given by

$$\boldsymbol{\theta = \frac{1}{2}\tan^{-1}\left(\frac{2\tau_{xy}}{\sigma_x - \sigma_y}\right)}$$

$$\theta = \frac{1}{2}\tan^{-1}\left(\frac{2\times 45}{80 - 0}\right)$$

$$\boldsymbol{\theta = 24^{\circ}11'\ and\ \theta = 114^{\circ}11'}$$

The first value of θ corresponds to σ_I while the second value corresponds to σ_{II}

The maximum shear stress is obtained from

$$\boldsymbol{\tau_{max} = \left(\frac{\sigma_I - \sigma_{II}}{2}\right)}$$

$$\tau_{max} = \left(\frac{100.2 - (-20.2)}{2}\right) = \mathbf{60.2\ N/mm^2}$$

and will act on planes at 45^0 to the principal planes.

NOTE: For $\sigma_x = -\sigma_y$, plane stress problem can be converted into equivalent plane strain problem and vice versa.

--------------------------------Pages missing in sample copy----------------------------------
----

1.17 MOHR'S CIRCLE

It is the graphical representation of stresses.

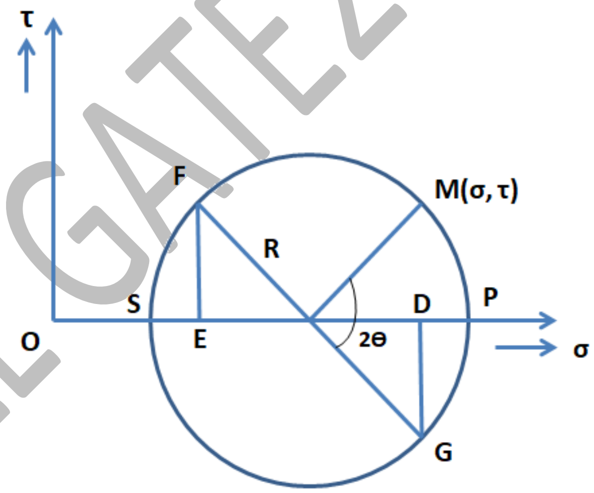
OD = σ_x

OE = σ_y

EF = DG = τ_{xy}

OP = Major Principal Stress

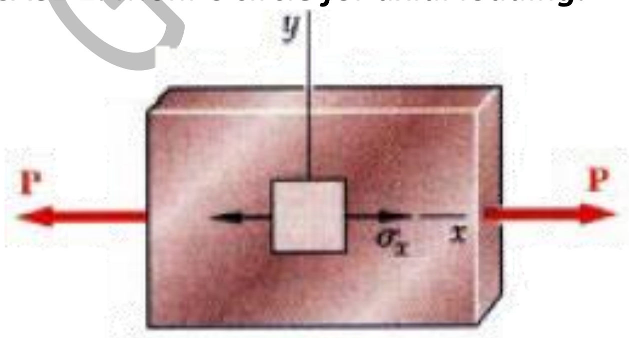
OS = Minor Principal Stress

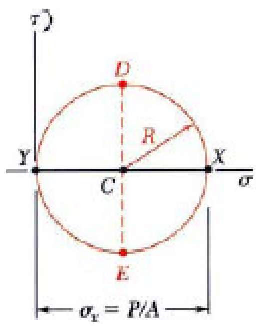


$$Radius\ of\ Mohr'circle, R = \sqrt{\left(\frac{\sigma_{xx} - \sigma_{yy}}{2}\right)^2 + \tau_{xy}^2}$$

$$= \tau_{max}$$

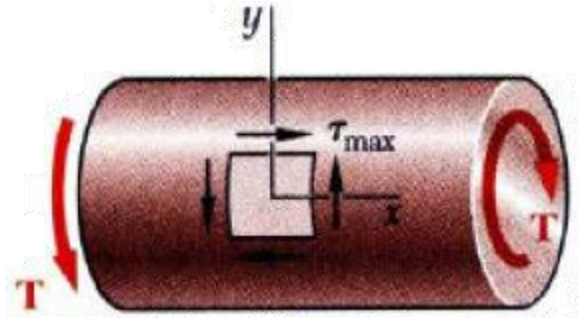
CASE 1: Mohr's circle for axial loading:

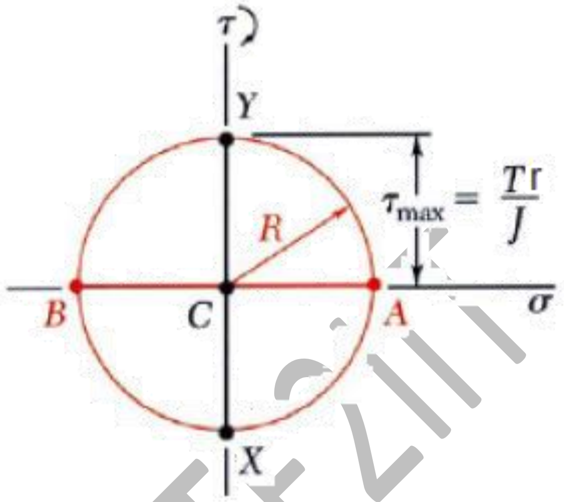




$\sigma_x = \frac{P}{A}, \sigma_y = \tau_{xy} = 0$

CASE 2: Mohr's circle for torsional loading:





$\tau_{xy} = \frac{Tr}{J}, \sigma_y = \sigma_x = 0$

It is a case of pure shear

CASE 3: A shaft compressed all round by a hub

--------------------------------Pages missing in sample copy-----------------------------------
----

Volumetric Strain (Dilation)

$$\varepsilon_I = longitudnal\ strain = \frac{\sigma_1}{E} - \mu\frac{\sigma_2}{E} = \frac{\text{pd}}{4\text{tE}}[1-2\mu]$$

$$\varepsilon_{II} = \frac{\sigma_2}{E} - \mu\frac{\sigma_1}{E} = \frac{\text{pd}}{4\text{tE}}[1-2\mu]$$

Volumetric Strain (e) $= \frac{\Delta V}{V} = \boldsymbol{\varepsilon_I + 2\varepsilon_{II}} = \frac{\textbf{pd}}{\textbf{4tE}}[\mathbf{5-4\mu}]$

i.e. Volumetric strain(e) = longitudinal strain(ε_I) + 2 x circumferential strain(ε_{II})

Question 4: A cylindrical shell has the following dimensions: Length = 3 m, Inside diameter = 1 m, Thickness of metal = 10 mm, Internal pressure = 1.5 MPa. Calculate the change in dimensions of the shell and the maximum intensity of shear stress induced. Take E = 200 GPa and Poisson's ratio v=0.3.

Solution:

Hoop stress or circumferential stress = $\sigma_1 = \frac{pd}{2t} = \frac{1.5\times1\times10^3}{2\times10} = 75\ MPa$

Longitudinal stress = $\sigma_2 = \frac{pd}{4t} = \frac{1.5\times1\times10^3}{4\times10} = 37.5\ MPa$

Hoop strain (ε_1) = $\frac{1}{E}(\sigma_1 - \mu\sigma_2) = 0.31875 \times 10^{-3}$

Change in diameter, $\Delta d = \varepsilon_1 \times d = 0.31875 \times 10^{-3} \times 1000 = 0.31875\ mm$

Longitudinal strain (ε_2) = $\frac{1}{E}(\sigma_2 - \mu\sigma_1) = 7.5 \times 10^{-5}$

Change in length, $\Delta l = \varepsilon_2 \times l = 7.5 \times 10^{-5} \times 3000 = 0.225\ mm$

Maximum shear stress,

$\tau_{max} = \frac{pd}{8t} = \mathbf{18.75\ MPa\ (in\ plane)}$

$\tau_{max} = \frac{pd}{4t} = \mathbf{37.5\ MPa\ (out\ of\ plane)}$

--------------------------------Pages missing in sample copy-----------------------------------

----

1.20 AIRY STRESS FUNCTION

- It can be used only for 2D problems.

$$\sigma_x = \frac{\partial^2\Phi}{\partial y^2}, \quad \sigma_y = \frac{\partial^2\Phi}{\partial x^2}, \quad \sigma_{xy} = -\frac{\partial^2\Phi}{\partial x\partial y}$$

- Stress equation of compatibility in 2D ,

$$\nabla^2(\sigma_x + \sigma_y) = 0$$
$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)(\sigma_x + \sigma_y) = 0$$

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)\left(\frac{\partial^2 \Phi}{\partial y^2} + \frac{\partial^2 \Phi}{\partial x^2}\right) = 0$$

$$\frac{\partial^4 \Phi}{\partial x^4} + 2\,\frac{\partial^4 \Phi}{\partial x^2\, \partial y^2} + \frac{\partial^4 \Phi}{\partial y^4} = 0$$

$$\nabla^4 \Phi = 0$$

- The Airy stress function is bi-harmonic.
- Airy stress function which satisfies the bi-harmonic equation will satisfy equilibrium and corresponds to compatible strain fields.

1.21 **THERMAL OR TEMPERATURE STRESS AND STRAIN**

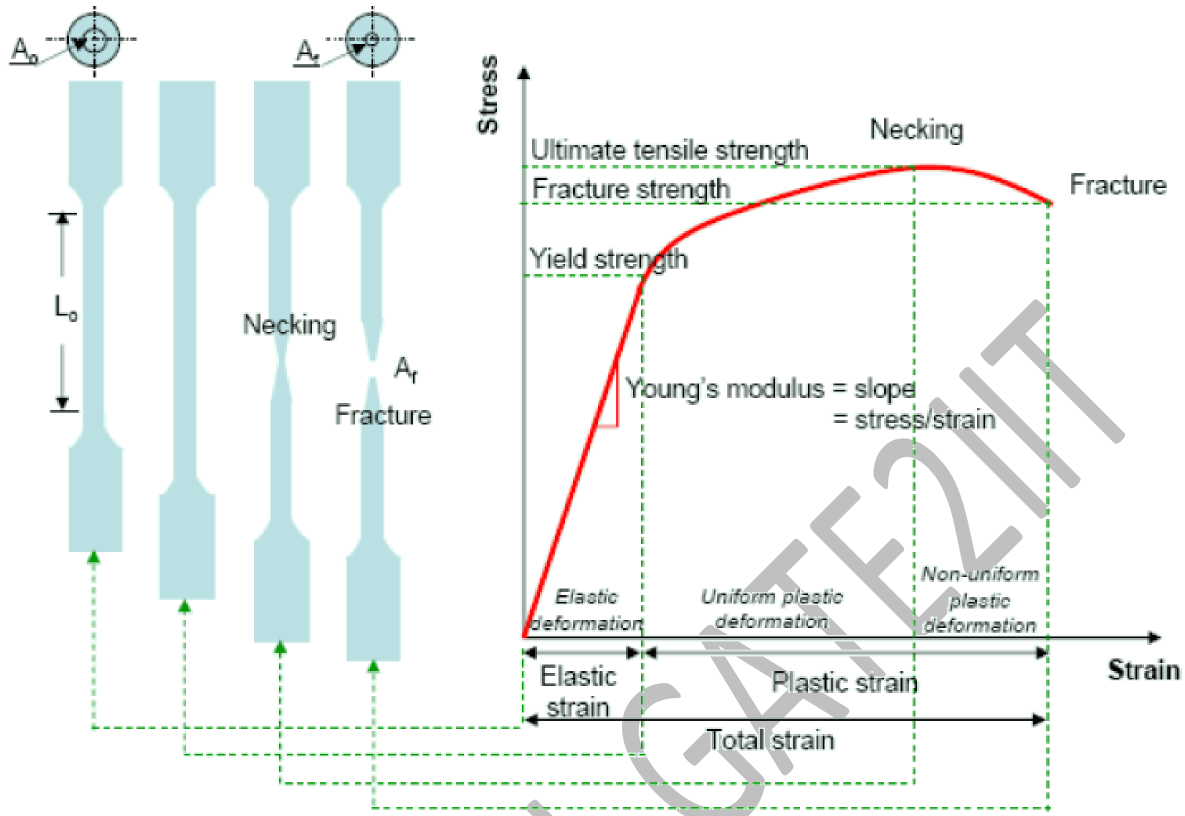
- When a material undergoes a change in temperature, it either elongates or contracts depending upon whether temperature is increased or decreased of the material.
- If the elongation or contraction is not restricted, i. e. free then the material does not experience any stress despite the fact that it undergoes a strain.
- The strain due to temperature change is called thermal strain and is expressed as,

$$\boldsymbol{\varepsilon} = \boldsymbol{\alpha}(\Delta \mathbf{T})$$

- Where α is co-efficient of thermal expansion, a material property, and ΔT is the change in temperature.
- The free expansion or contraction of materials, when restrained induces stress in the material and it is referred to as thermal stress.

-----------------------------------Pages missing in sample copy------------------------------------
----

1.22 TENSILE TEST



- **True elastic limit:** based on micro-strain measurement at strains on order of 2 × 10-6. Very low value and is related to the motion of a few hundred dislocations.

- **Proportional limit:** the highest stress at which stress is directly proportional to strain.

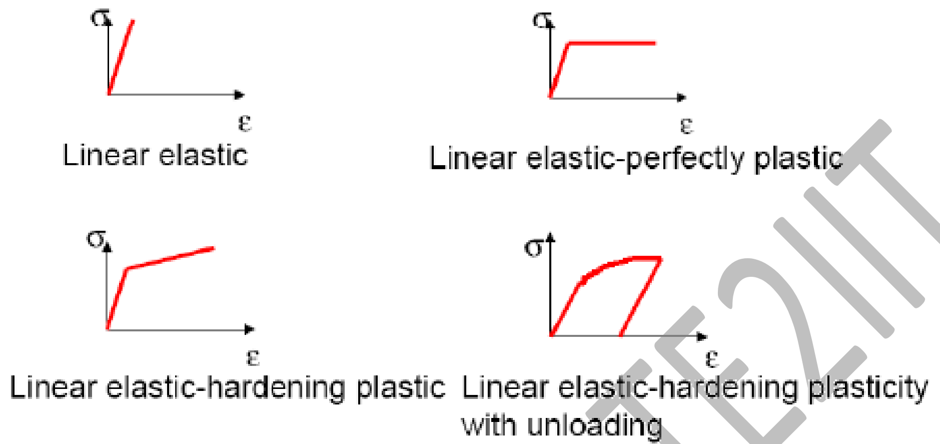
- **Elastic limit:** is the greatest stress the material can withstand without any measurable permanent strain after unloading. Elastic limit > proportional limit.

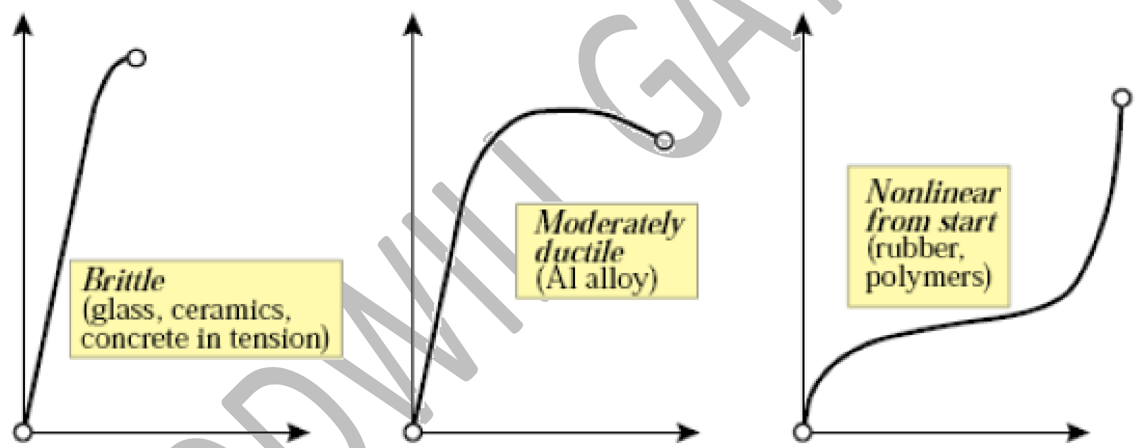
- **Yield strength:** is the stress required to produce a small specific amount of deformation. The offset yield strength can be determined by the stress corresponding to the intersection of the stress-strain curve and a line parallel to the elastic line offset by a strain of 0.2 or 0.1%.(ε = 0.002 or 0.001).
  The offset yield stress is referred to proof stress either at 0.1 or 0.5% strain used for design and specification purposes to avoid the practical difficulties of measuring the elastic limit or proportional limit.

- **Tensile strength or ultimate tensile strength (UTS)**
  σ_u is the maximum load Pmax divided by the original cross-sectional area A_0 of the specimen.

- **Modulus of Elasticity or Young's Modulus**
  It is slope of elastic line upto proportional limit.

- **Stress-strain response**





GATE and Additional objective questions

Q1. Which of the following statement(s) is/are true about the state of a body in plane strain condition? **[GATE 2018]**

P: All the points in the body undergo displacements in one plane only, for example the x-y plane, leading to $\varepsilon_{zz} = \gamma_{yz} = \gamma_{xz} = 0$.
Q: All the components of stress perpendicular to the plane of deformation, for example the x-y plane of the body are equal to zero .i.e. $\sigma_{zz} = \tau_{xz} = \tau_{yz} = 0$.
R: Except the normal component, all the other components of stress perpendicular to the plane of deformation of the body, for example the x-y plane, are equal to zero. i.e. $\sigma_{zz} \neq 0, \tau_{xz} = \tau_{yz} = 0$.

(a) P only (b) Q only (c) P and Q (d) P and R

Ans: (d)

Plane Strain

- For a plane strain problem, strain components perpendicular to the plane of plane strain condition are zero.
- If xy is the plane of plane strain problem then

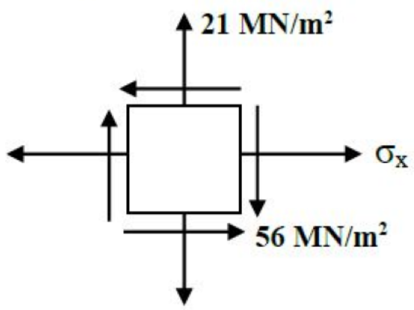
Non-zero strain components are $\varepsilon_x, \varepsilon_y, \Upsilon_{xy}$

Zero strain components are $\varepsilon_z, \Upsilon_{xz}, \Upsilon_{yz}$

Non-zero stress components are $\sigma_x, \sigma_y, \sigma_z$ and $\boldsymbol{\tau_{xy}}$

Zero stress components are $\boldsymbol{\tau_{xz}}, \boldsymbol{\tau_{yz}}$

Q2. For the state of plane stress shown in the figure, the minimum principal stress is -7 MN/m^2. The normal stress σ_x in MN/m^2 is equal to __________ (round to nearest integer). **[GATE 2017]**



--------------------------------Pages missing in sample copy----------------------------------
----

EXERCISE

Q1. The normal stress on a a plane whose normal is inclined at angle Θ with the line of action of the uniaxial stress σ_x is given by

(a) $\frac{\sigma_x}{\cos^2\theta}$ **(b)** $\frac{\sigma_x}{\sin^2\theta}$ **(c)** $\sigma_x \cos^2\theta$ **(d)** $\sigma_x \sin^2\theta$

Q2. If one of the principal stresses at a point is zero, the magnitude of the other principal stress must be ______of the maximum shear stress.

(a) Equal to (b) one and a half times (c) twice (d) two and a half times

Q3. In the case of pure shear, the principal stresses are

(a) Equal in magnitude to the shear stress and similar in nature

(b) Equal in magnitude to the shear stress and opposite in nature

(c) Equal in magnitude to half the maximum shear stress and similar in nature

(d) Equal in magnitude to half the maximum shear stress and opposite in nature

Q4. The following state of plain strain condition is observed in a rectangular coordinate system

$$\varepsilon_x = p(x^2 + y^2), \varepsilon_y = p(x^2 + y^2), \gamma_{xy} = qxy$$

The strain is compatible if:

(a) p = 2q (b) 2p = q (c) p = 4q (d) 4p = q

--------------------------------Pages missing in sample copy----------------------------------
----

END of Sample copy